Feb/7

$(qs^{-1}, q)_\infty \; \frac{1}{(s, q)_\infty}$

$s = xq$

$x^{-1} q^{-1}$

$(x^{-1}, q)_\infty \quad \frac{1}{(xq, q)_\infty}$

$x = \frac{Q}{t}$

$Q = xt$

$(x, t^{-1})_\infty \cdot (xt, t)_\infty = 1$

$(x, q^{-1})_\infty = \frac{1}{(xq, q)_\infty}$

$$\frac{(xq, q)^D_\infty}{(x^{-1}, q)^N_\infty} = \frac{1}{(x^{-1}, q)_\infty \, (x, q^{-1})_\infty}$$

$(qs^{-1}, q)^D_\infty \longrightarrow$

$= (s^{-1}, q^{-1})^{-1}_\infty \quad \frac{\tilde{s} = s}{\tilde{q} = q} \quad (\tilde{s}, \tilde{q})^{-1}_\infty$

$x = s^{-1}$

How to interprete the $\Theta(\cdot, \cdot)(\cdot, \cdot)$ in (2d susy term) $\Theta(\;,\;)(\;)$ term of 3d brane webs ?

$\theta(-\sqrt{q}\,x)\;\Theta = (qx; q)_\infty (x^{-1}; q)_\infty$

$$(x, q; q)^D_\infty = \frac{\theta(-\sqrt{q}\,x)}{\cancel{(x^{-1}; q)_\infty}} \quad (x^{-1}; q)^{-1\,N}_\infty$$

chiral

chiral.

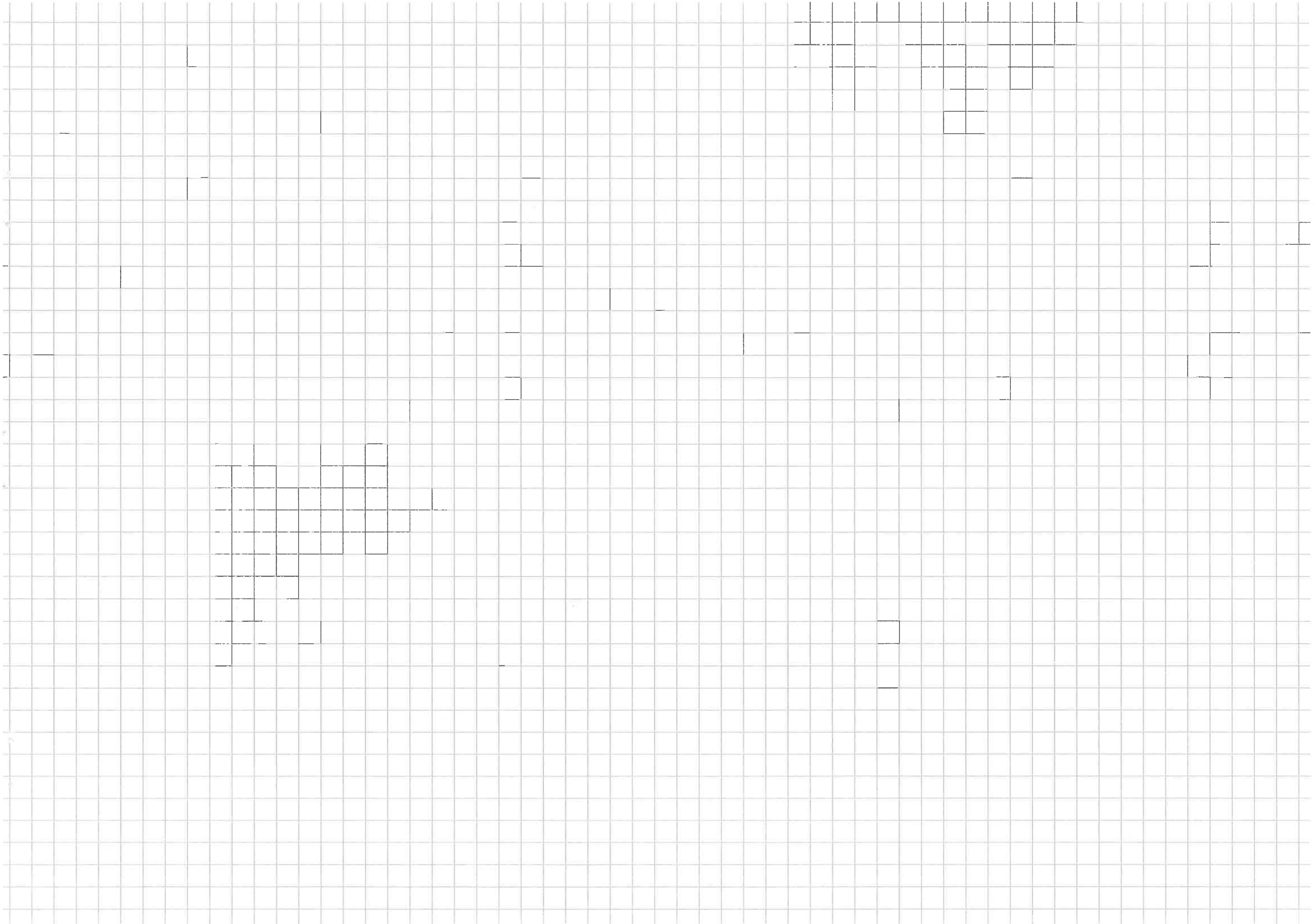
q

chiral

$D \xrightarrow{\theta(-\sqrt{q}\,x)} N$

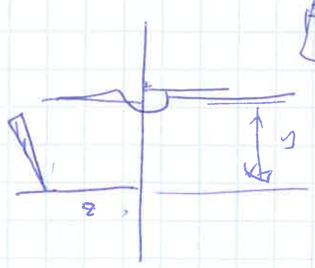
? n T-transf.

as in T-transf.



Feb 13

$$= \frac{i\pi\Omega^2}{8} + \frac{\pi\Omega}{2}\cdot\frac{1}{\dots} - \frac{\pi i}{2} z^2$$



$$\boxed{F(z) = \oint S_b\left(\frac{i\Omega}{2} - x\right) S_b\left(\frac{i\Omega}{2} + x\right) e^{-2\pi i z x}\, dx} = S_b\left(\frac{i\Omega}{2}\right) \quad (*)$$

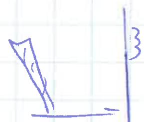
(prove we do this by ...)

(gluing ... origin)

$$\boxed{m(F) = m(AF)} = y = 0, \qquad e^{-\frac{\pi i}{2}\left(\frac{i\Omega}{2}+z\right)^2} S_b\left(\frac{i\Omega}{2}+z\right) \ (2)$$

Which one is correct?

or $\boxed{m(F) \simeq -m(AF)}$ (Higgsing) (decoupling) (1), (2)



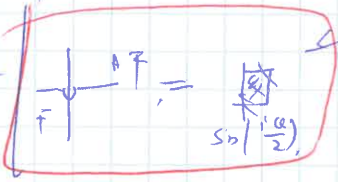
decoupling

$\exists\ S_b\left(\frac{i\Omega}{2}\right)$

free-chiral,

$$\frac{\partial}{\partial z} F(z) = 0, \qquad \frac{\partial F(z)}{\partial z} = \oint S_b\left(\frac{i\Omega}{2} - x\right) S_b\left(\frac{i\Omega}{2} + x\right) e^{-2\pi i z x} (-2\pi i x)\, dx$$

$$\boxed{S_b\left(\frac{i\Omega}{2} - z\right) S_b\left(\frac{i\Omega}{2} + z\right) = S_b\left(\frac{i\Omega}{2}\right) \oint e^{-2\pi i z x}\, dx} \quad (*)$$



ω_j

$$S_b\left(\frac{i\Omega}{2}\right) = \prod_{m,n} \frac{mb + \frac{n}{b} + Q}{mb + \frac{n}{b}}$$

$$+ \frac{i\pi}{2}(+)\frac{Q^2}{4}, \quad \frac{\ }{8}$$

$$\mathfrak{q} = e^{2\pi i b^2}$$

$$-\frac{i}{\ }\frac{i\Omega}{2} = \frac{\Omega}{2}$$

$$\Rightarrow e^{-\frac{i\pi}{2}\left(\frac{i\Omega}{2}\right)^2} PE\left[\frac{1}{1-\mathfrak{q}^{\dots}}\right]$$

$$= e^{\frac{\pi i \Omega^2}{8}} PE\left[\frac{-\mathfrak{q}}{1-\mathfrak{q}}\right] = (\mathfrak{q}, \mathfrak{q})_\infty \qquad \frac{\mathfrak{q}}{\mathfrak{q}-1}$$

$\boxed{z=0}$ $\oint \cancel{S_b(y)\, S_b(y)\, e^{-2\pi i z x}\, dz} =$

$$S_b\left(\frac{i\Omega}{2}\right) S_b(y)\, S_b(y) = \oint S_b\left(\frac{i\Omega}{2} - x - y\right) S_b\left(\frac{i\Omega}{2} + x - y\right) dx,$$

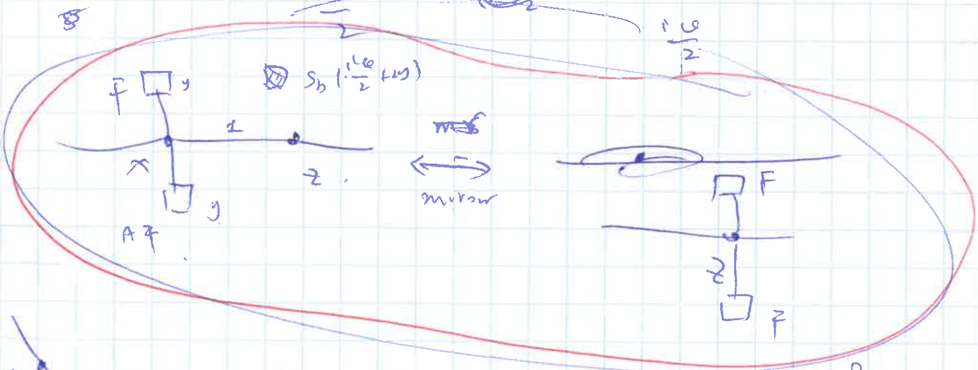
$S_b\left(\frac{i\Omega}{2}\right) \to \delta(z, y)$

$$y = \frac{i\Omega}{2} + u$$

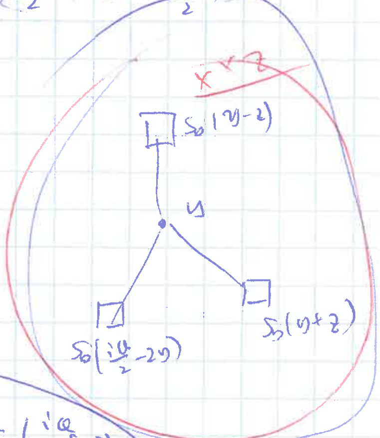
$$\int dz\, S_b\left(\frac{i\Omega}{2} - z + u\right) S_b\left(\frac{i\Omega}{2} + z + u\right) = \boxed{S_b\left(\frac{i\Omega}{2} + 2u\right)} \cdot \int dz \int dx\, e^{-2\pi i z x},\ S_b\left(\frac{i\Omega}{2} - x - y\right)$$

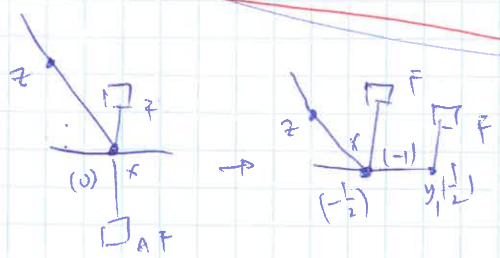
$\frac{i\Omega}{2} + 2u - \frac{i\Omega}{2}$

$\frac{i\Omega}{2}$



$\frac{i\Omega}{2} - x - \frac{i\Omega}{2} \to u$





$$\left[(+)^2, \left(\frac{i\Omega}{2} + z\right)\right]^2$$

$$-\frac{\pi i}{\ }$$

$$e^{\frac{\pi i}{8}(\Omega - 2iz)^2} S_b\left(\frac{i\Omega}{2} + z\right)$$

$$= e^{-\frac{\pi i}{2}\left(\frac{i\Omega}{2}+z\right)^2} S_b\left(\frac{i\Omega}{2} + z\right)$$

gamma reduction

$2i \cdot \frac{\Omega}{2}$

$\Omega \cdot \frac{2}{i}$

$d = a_0 r_i$

$a_0 = a(t_0)$

$\sim \frac{1}{q^2}$

light shift by $1+z$

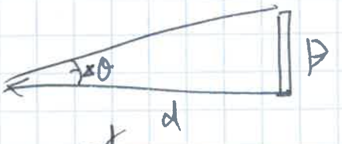
problem 2.1

Luminosity distance

$d_L = a_0 r_i (1+z)$

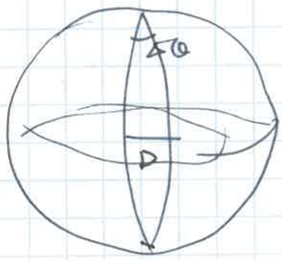
~~problem 2~~

problem 2.3



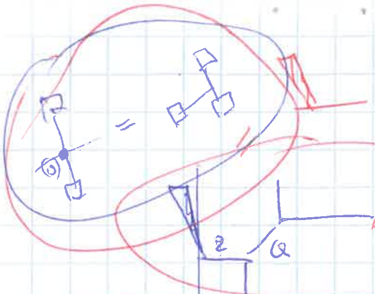
$\cancel{d = D\Delta\theta}$

$D = d\,\Delta\theta$



Feb 15

Feb 13



$m(F) \simeq m(AF) = 0 = 0$

$$z_{f\text{-}br} = \frac{(z\sqrt{tq}, q)_\infty}{(zQt, q)_\infty} \begin{matrix} \leftarrow F \\ \leftarrow AF \end{matrix} \overset{t=q}{=} \frac{(zq, q)_\infty}{(zQq, q)_\infty} \overset{Q=1}{=} 1$$

$$\boxed{\frac{1}{(Q,q)_\infty} \sum_{n\geq 0} z^n \frac{(Q,q)_n}{(q,q)_n} = \frac{(zq, q)_\infty}{(zQq, q)_\infty}}$$

$\sqrt{tq} = Qt \;\rightarrow\; Q = \sqrt{\frac{q}{t}}$

$z = \frac{\tilde{z}}{\sqrt{q}}$

$x = \sqrt{q}\,\tilde{x}$

$x^{-1} = q^{-\frac{1}{2}}\tilde{x}^{-1}$

$$\frac{(z, q)}{(zd, q)}$$

$$\boxed{\theta(-\sqrt{q}\,x, q) = (qx, q)_\infty (x^{-1}, q)_\infty}$$

What is this dery. ...

NOT Strange

$\rightsquigarrow \; (Q_1\sqrt{\tfrac{t}{q}}, t)_n (Q_2^{-1}\sqrt{\tfrac{t}{q}}, t)_n$

what strange

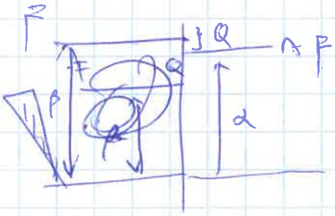
$\boxed{Q_1 Q_2 = q Q_2}$

$\overset{t=q}{=} (Q_1, q)_n (Q_2^{-1}, q)_n$

$\boxed{Q_1 = q Q_2}$

$\boxed{Q_1(-\sqrt{q}\cdot d_2, q)}$

$\boxed{\alpha_1 \alpha_2^{-1} = q}$



$$\rightsquigarrow \frac{(\alpha, q)_n}{(\beta, q)_n} \begin{matrix} \leftarrow AF \\ \leftarrow F \end{matrix}$$

$\boxed{\beta = \alpha Q}$

$$\frac{(\alpha\sqrt{\tfrac{t}{q}}, t)_n}{(\alpha Q\sqrt{\tfrac{t}{q}}, t)_n} = 1 \Rightarrow \boxed{Q = \sqrt{\frac{q}{t}}}$$

$\alpha\sqrt{\tfrac{t}{q}} = \alpha Q\sqrt{\tfrac{t}{q}}$

$Q_1, \; \alpha_1 Q$

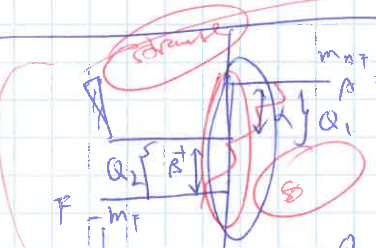
$\tilde{z} = z + \frac{m}{2}$

$Q_1^2 Q = q$

$\tilde{z} = z + \frac{m}{2}$

$Q = q Q_1^{-2}$

$\alpha \sim e^m, \; \beta \sim e^m$



strange

$$\rightsquigarrow \frac{(\alpha\sqrt{\tfrac{t}{q}}, t)_n}{(\beta\tfrac{t}{q}, t)_n} \begin{matrix} \leftarrow AF \\ \leftarrow F \end{matrix} = 1 \Rightarrow \boxed{\alpha\beta^{-1} = \sqrt{\tfrac{t}{q}}}$$

$Q\sqrt{\tfrac{t}{q}} = Q_2^{-1}\tfrac{t}{q}$

$\boxed{Q_1 Q_2 = \sqrt{\tfrac{t}{q}}}$

strange

$\alpha \simeq \beta$

$m(F) \; m(AF)$

$\alpha\sqrt{\tfrac{t}{q}} = \beta^{-1}\sqrt{\tfrac{t}{q}} \Rightarrow \alpha\beta$

$\alpha = Q_1$

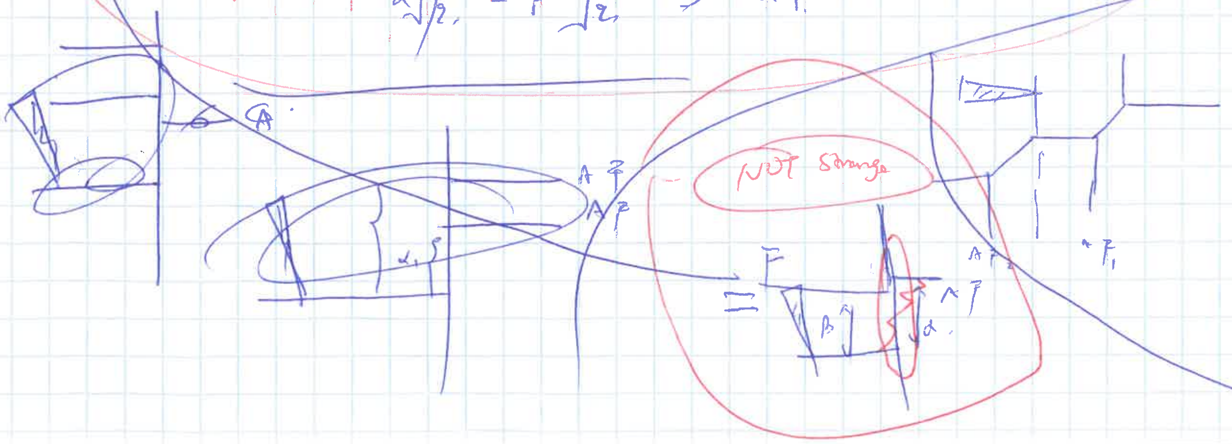
$\beta = Q_2^{-1}$

$\sim Q_1$

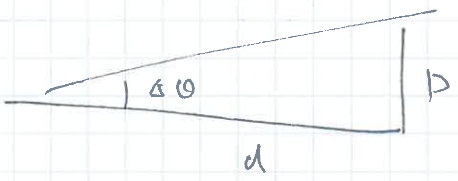
$\beta^{-1} = Q_2$

$Q_1 Q_2 = 1$

$Q_1^{-1} \sim Q$



NOT Strange



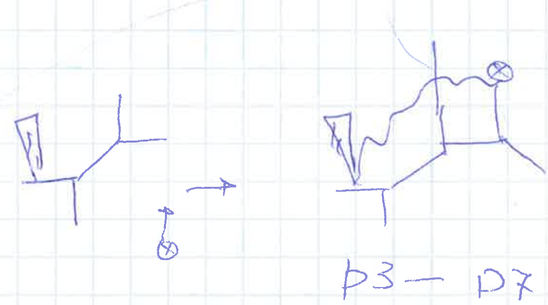
for flat space

~~in flat space~~

$$\Delta\theta = \frac{D}{d_A}$$

$\uparrow$ $\cancel{a(t_1)\, r_1}$

$$d_A = a(t_1)\, r_1$$



$$\Omega_i = \frac{\rho_i(t)}{\rho_{cr}(t)}, \qquad \Omega_R(z) = \Omega_R(z=0)\,(1+z)^{3/2}$$

2.16

$$d_r = \int_0^{r_2} \frac{dr}{\sqrt{1-kr^2}} = \frac{1}{a_0 H_0} \int_{(1+z)^{-1}}^{1} \frac{dx}{\sqrt{(1-\Omega_0)x^2 + \Omega_0 x}}$$

$$= \frac{1}{\boxed{a_0} H_0} \frac{1}{\sqrt{\Omega_0 - 1}} \arccos\left[1 - \frac{2(\Omega_0 - 1)}{\Omega_0(1+z)} \right] \Bigg|_{z_1 \to 3}^{z_0 \to 0 \text{ now}}$$

$$d(t_0) = a_0 d_r$$

$$d(a_{t_m}) = a(t_m)\, d_r = \frac{a(t_m)}{a(t_0)}\, d(t_0) = \frac{1}{1+z}\, d(t_m) = \frac{1}{4}\, d(t_m)$$

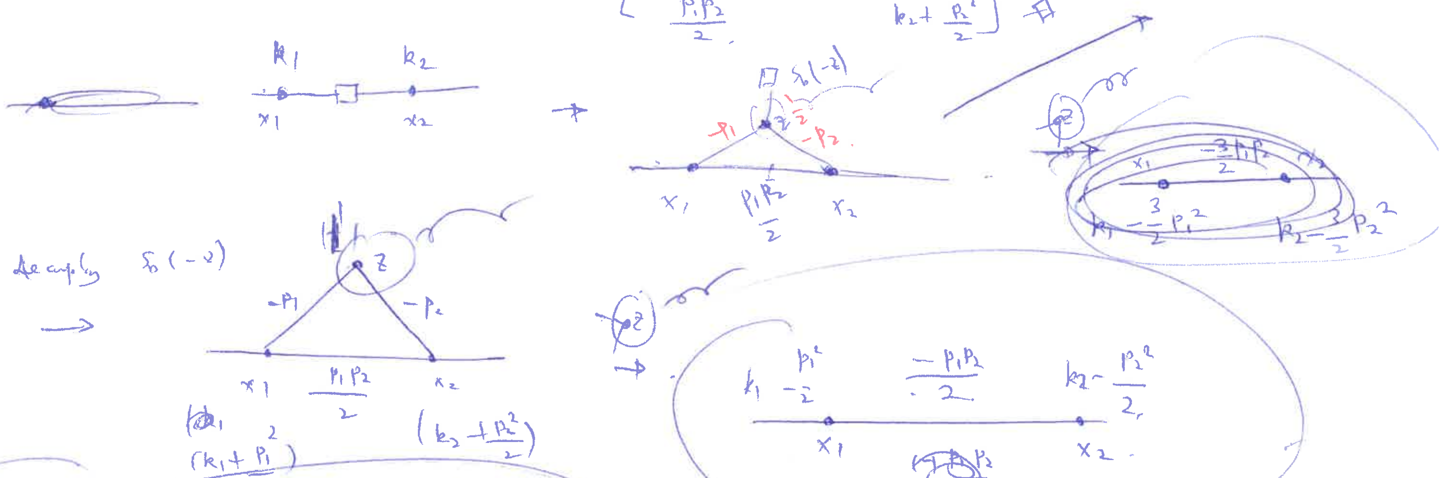
$$d(t_m) = \frac{1}{1+z_0}\, d(t_0)$$

In future. $H\tilde{z} = \dfrac{a_0}{\tilde{a}}$

Feb 27

Feb 25

$S_D(p_1x_1+p_2x_2) \to \begin{bmatrix} k_1+\frac{p_1^2}{2} & \frac{p_1p_2}{2} \\ \frac{p_1p_2}{2} & k_2+\frac{p_2^2}{2} \end{bmatrix}$



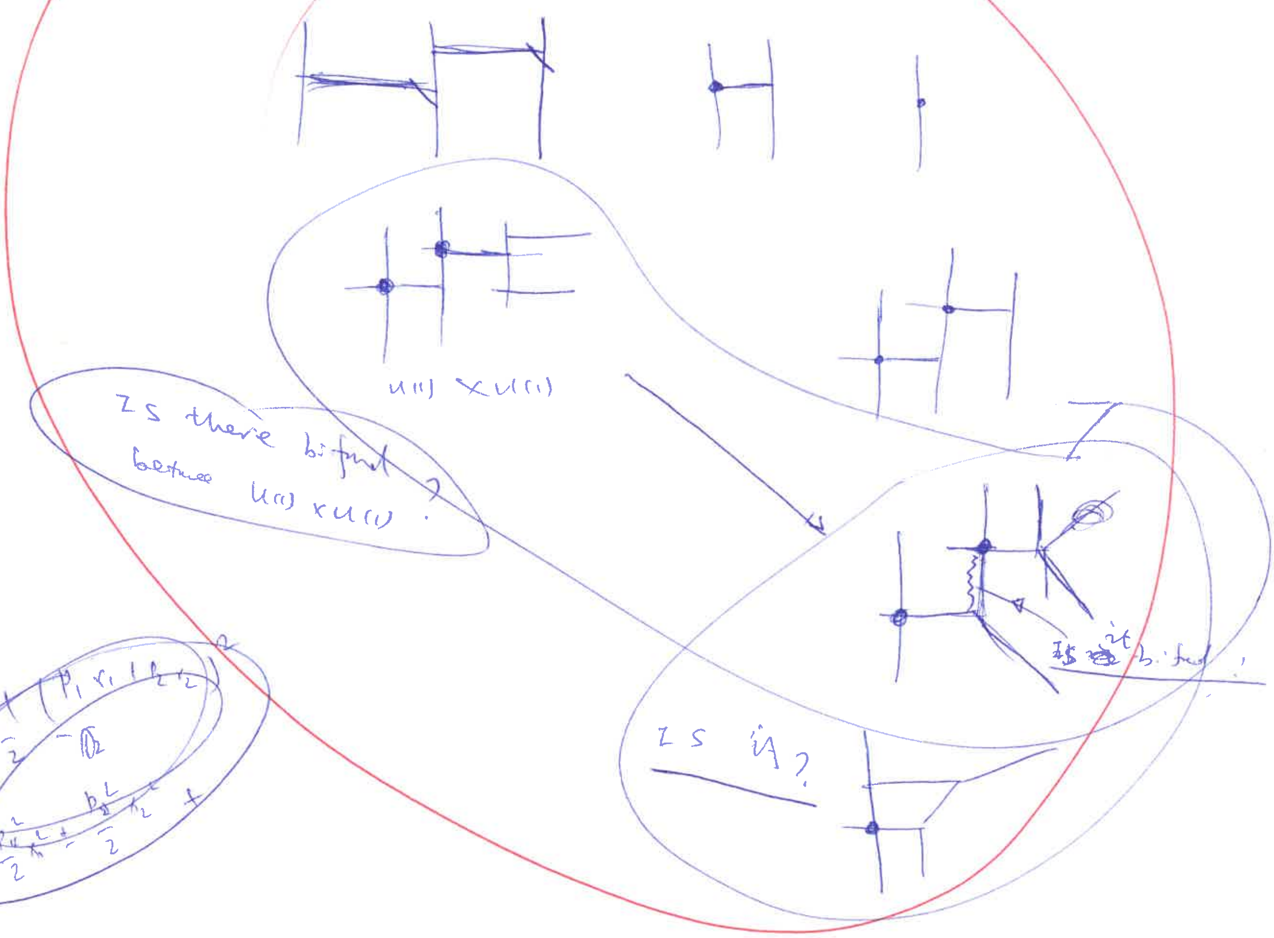
Should we have two bifund ?

if $p_1=-p_2=\pm\sqrt{2}$

k_1-1 ± 1 k_2-1

x_1 x_2

Brane web :



$(p_1x_1+p_2x_2)$

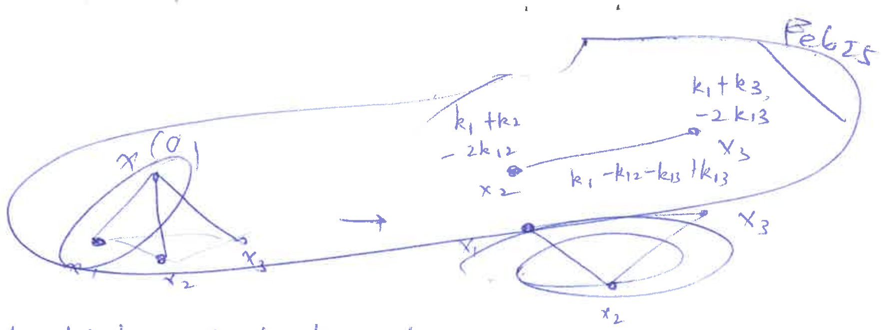
$k=1$

Feb 25

$\delta(x + x_1 + x_2 + \cdots)$

$x + x_1 + x_2 + \cdots = 0.$

$\sum_i x_i = 0$, $\cancel{x = -x_1 - x_2 - x_3 - \cdots}$ $x_1 + x_2 + x_3 = 0$



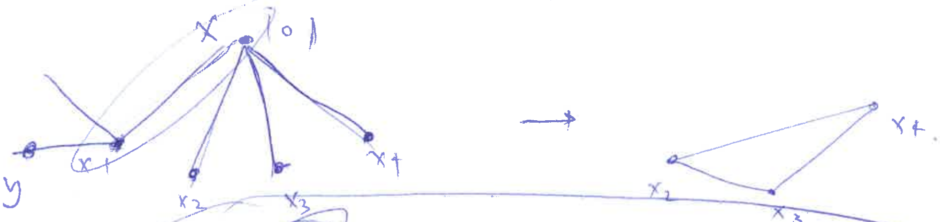
$$e^{k_1 x_1^2 + k_2 x_2^2 + k_3 x_3^2 + 2k_{12} x_1 x_2 + 2k_{23} x_2 x_3 + 2k_{13} x_1 x_3}$$

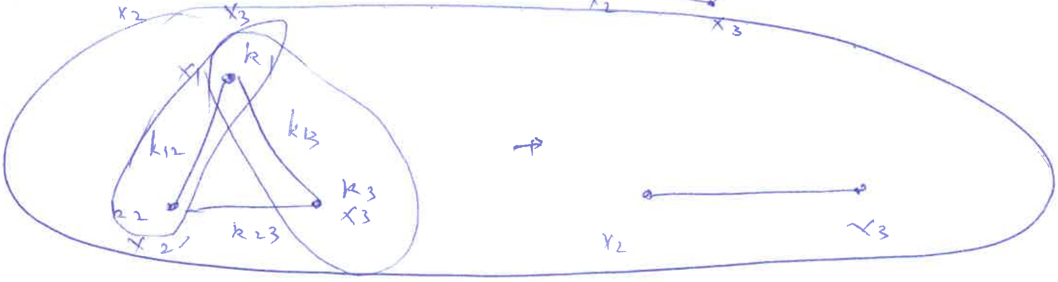
$$= e^{k_1 (x_2 + x_3)^2 + k_2 x_2^2 + k_3 x_3^2 + 2k_{12}(-x_2 - x_3) x_2 + 2k_{23} x_2 x_3 + 2k_{13}(-x_2 - x_3) x_3}$$

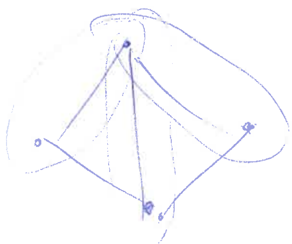
$$= e^{k_1 x_2^2 + k_1 x_3^2 + 2k_1 x_2 x_3 + \cdots - 2k_{12} x_2^2 - 2k_{12} x_2 x_3 + 2k_{23} x_2 x_3 - 2k_{13} x_2 x_3 - 2k_{13} x_3^2}$$

$$= e^{(k_1 + k_2 - 2k_{12}) x_2^2 + (k_1 + k_3 - 2k_{13}) x_3^2 + 2(k_1 - k_{12} + k_{23} - k_{13}) x_2 x_3}$$

$\cancel{-2k(x_1 + x_2 + x_3)}$ $\quad -2k(x_2 + x_3)$

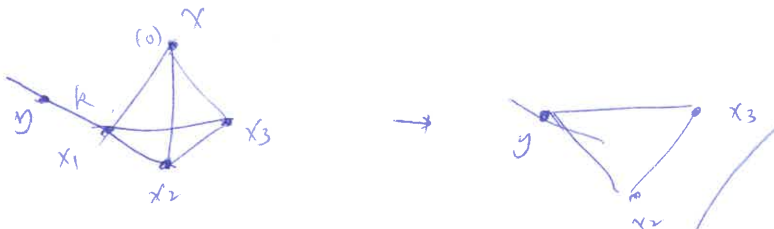


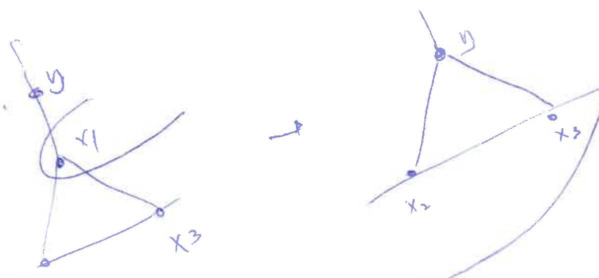




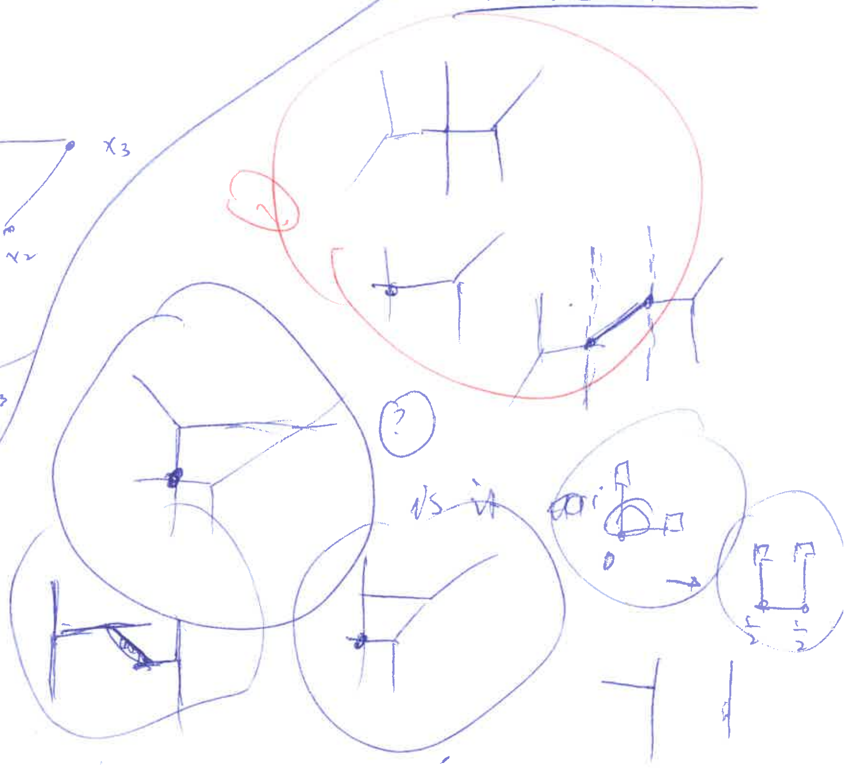
$\frac{1}{2} (2y)^2 \Rightarrow \frac{1}{2} 4 y^2$

$= 2y^2$



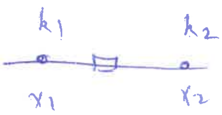


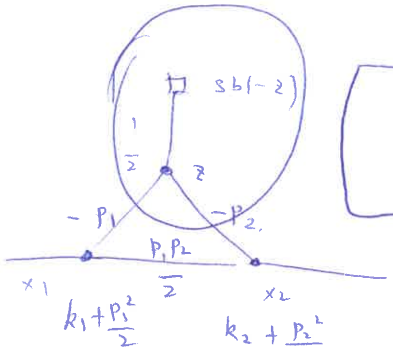
Brane Web :



Feb 27

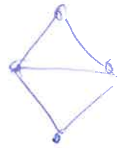
$S_b(p_1 x_1 + p_2 x_2)$



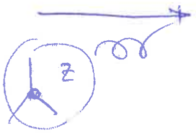


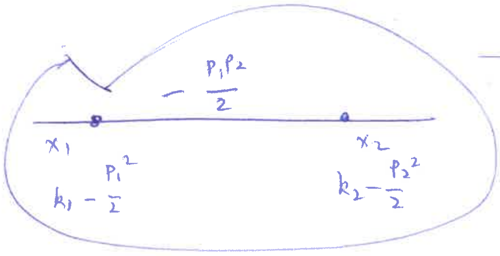
$\boxed{\text{Edge} = S}$

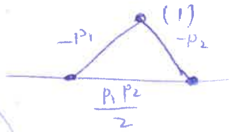
gauging



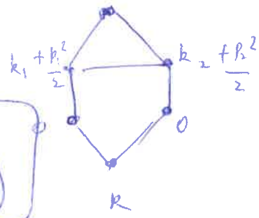
$S_b(-z) \to e^{(-)\frac{\pi i}{2} z^2}$



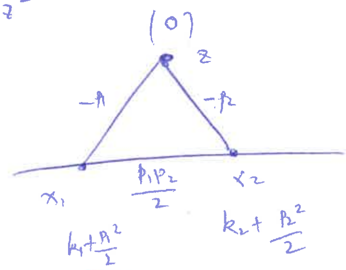


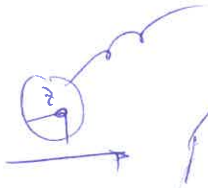


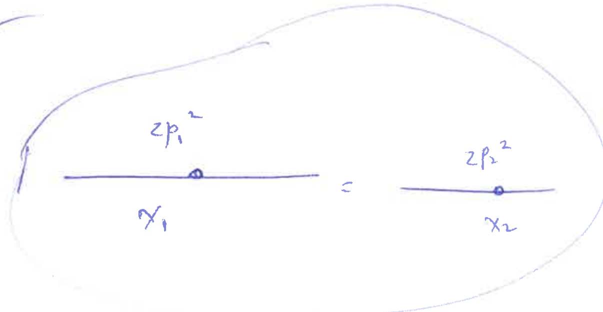
$\boxed{\text{Should we have two bifund ?}}$



$S_b(-z) \to e^{\frac{\pi i}{2} z^2}$



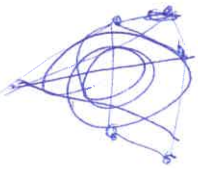


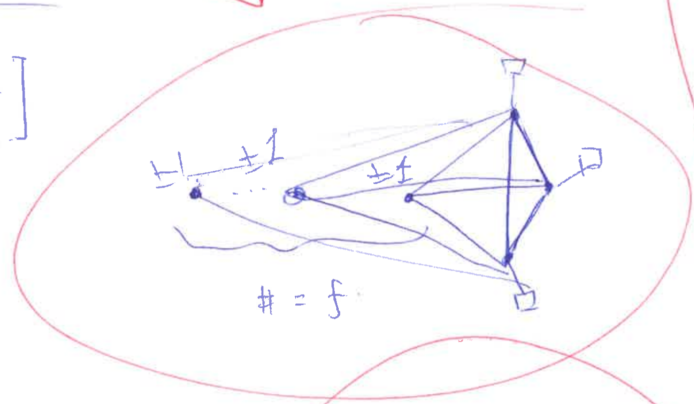


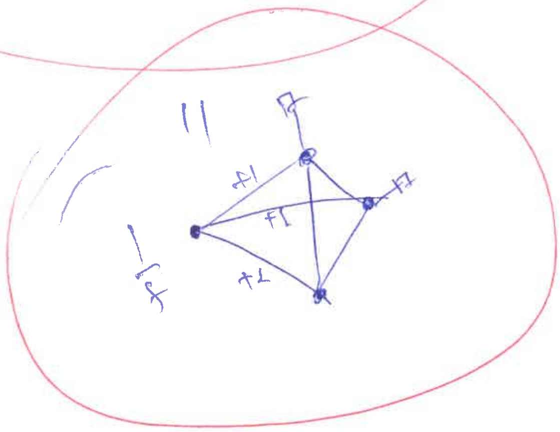
$\boxed{\text{Framing}}$

Framing # f

$e^{\pi i} \, k_{ij} \to k_{ij} + f \begin{bmatrix} 1 & 1 & 1 \\ 1 & \cdots & \cdots \\ 1 & \cdots & 1 \end{bmatrix}$







How to understand bifund. double using $S^{-}T$ operator ?

$\frac{2}{3} \times \frac{k}{3} = \frac{2}{9}$

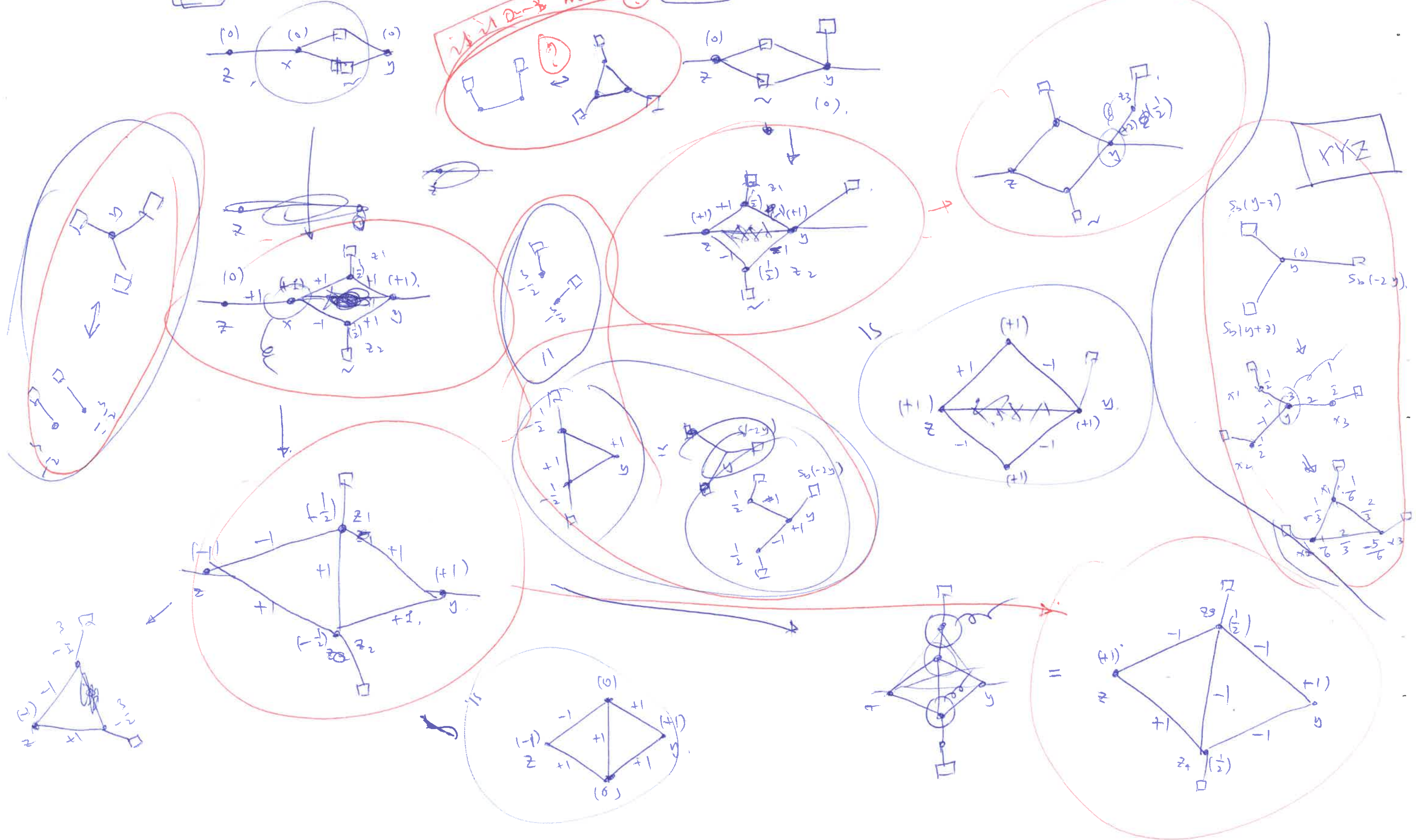
Apr 19

Feb 28

Feb 25.

$$\boxed{\int dz} \int dx \boxed{\int dy} \, e^{-2\pi i z x} \, S_b(-x-y)\, S_b(x-y) = \boxed{\int dz \int dy} \, S_b(y-z)\, S_b(y+z) \quad S_b(-2y)$$

is it 2-3 moves ?



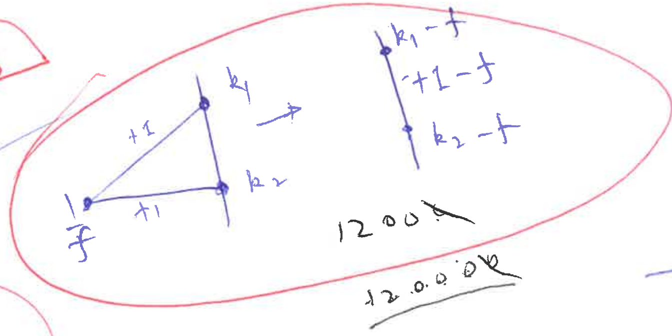
Feb 27

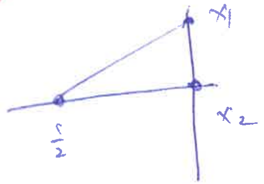
$S_b[-x]$ $\quad S_b[-x]$ $\quad S_b[\ldots]$ $\quad S_b[\frac{iQ}{4} \ldots]$

~~$S_b[-x]$ $S_b[x \ldots]$~~

framing

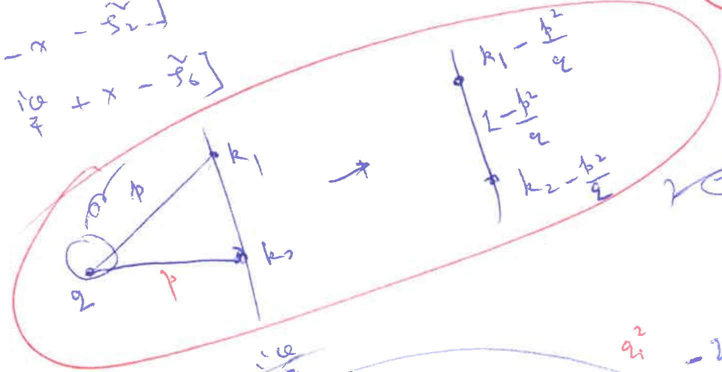
$f = 0, -1, -2$

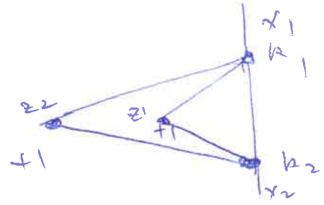




$S_b[\frac{iQ}{2} - x]$ $\quad S_b[\frac{iQ}{4} - x - \tilde{\xi}_2 \ldots]$

$S_b[\frac{iQ}{4} + x - \tilde{\xi}_3]$ $\quad S_b[\frac{iQ}{4} + x - \tilde{\xi}_6]$



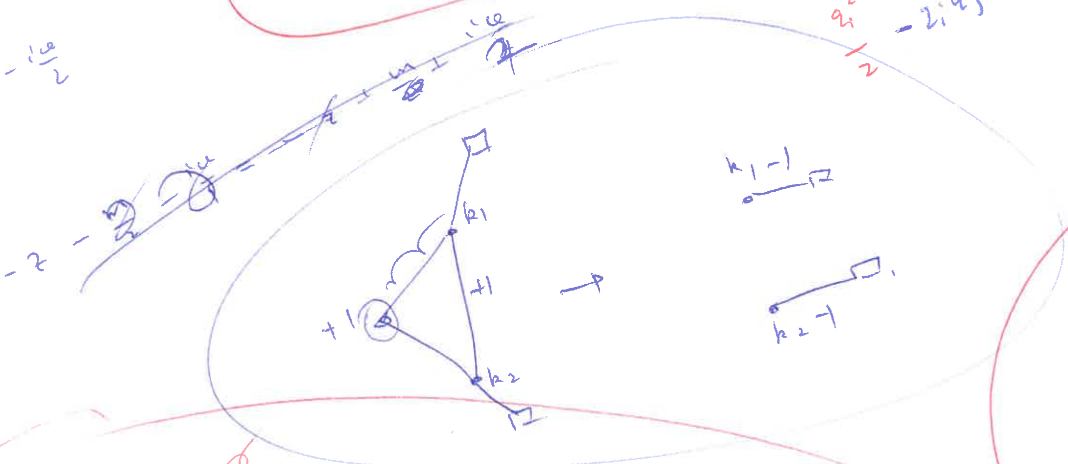


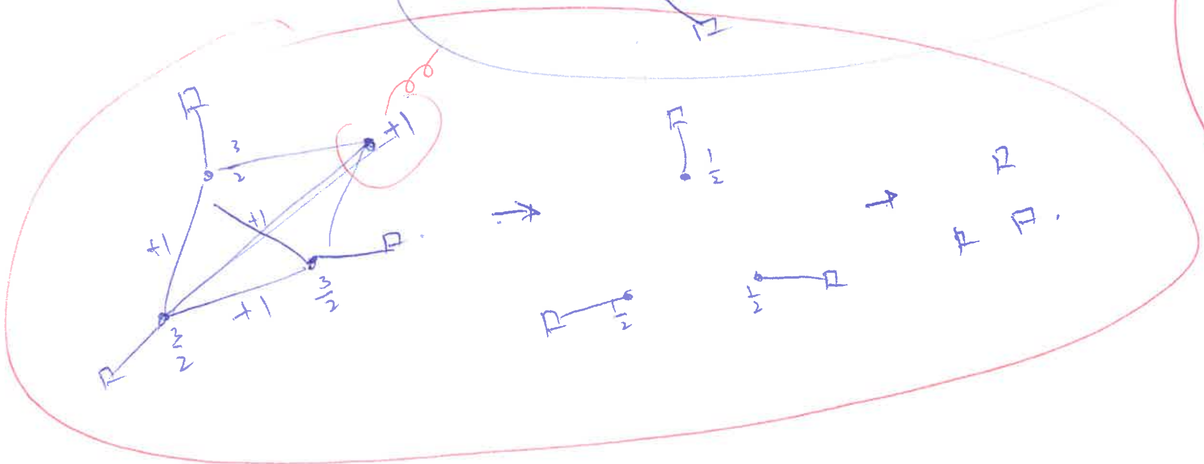
$2 z_1 x_1 + 2 z_1 x_2 + 2 z_2 x_1 + 2 z_2 x_2$

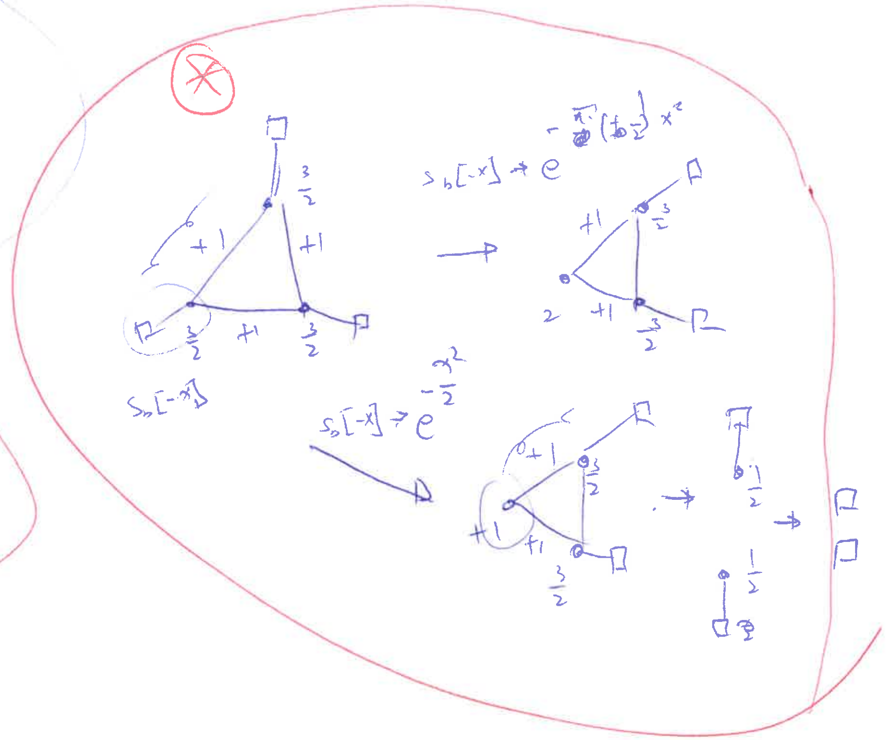
$2 x_1(z_1 + z_2) + 2 x_2(z_1 + z_2) + z_1^2 + z_2^2$

$\frac{q_i^2}{2} - z_i q_j$

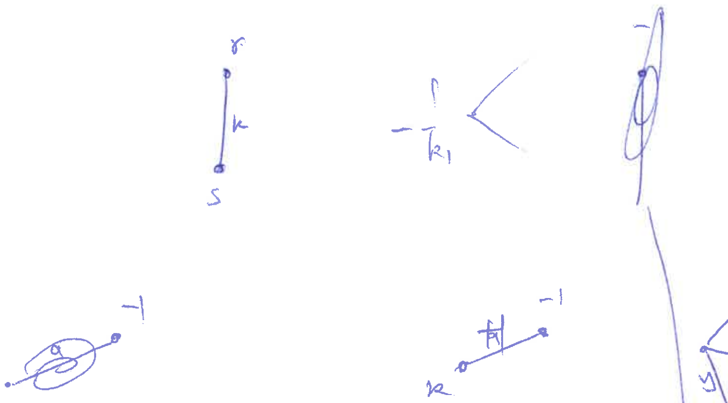
$m = -\frac{iQ}{2}$

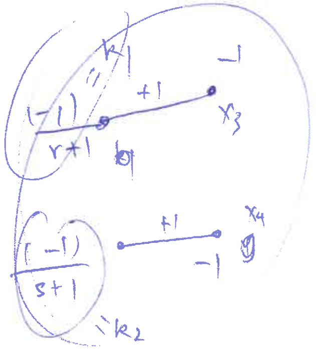


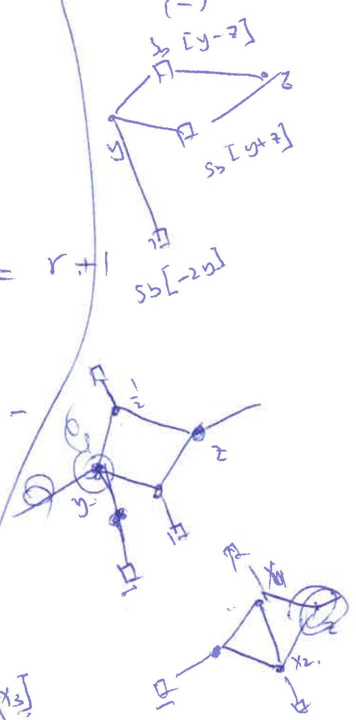




Mar/16

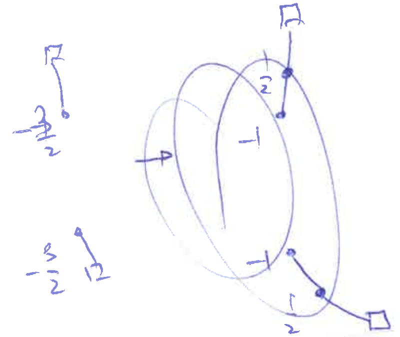


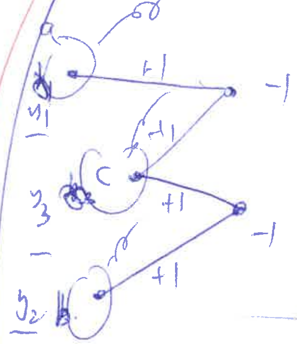


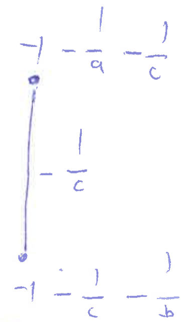


$\phi - \frac{k_2}{k} = r+1$

$\frac{1}{r+1}$







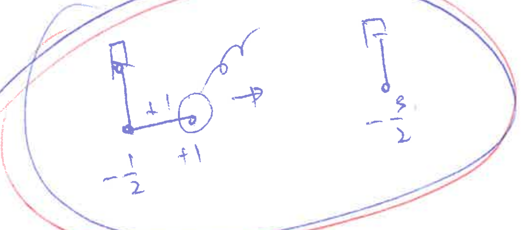
$c = -\frac{1}{k}$

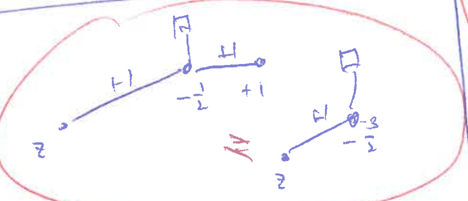
$a = \frac{1}{k-r-1}$

$b = \frac{1}{k-s-1}$

$$\int dx_1 e^{k_1^2 x_1^2}\, e^{2 i x_1 x_3}\, e^{-\frac{1}{2}x_3^2} \int dx_3\, S_b\left[\frac{ia}{2} - x_3\right] \quad e^{s x_3}$$

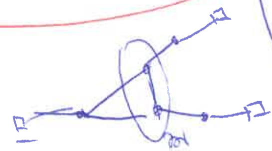
$$= \int dx_1\, e^{k_1^2 x_1^2} \int dx_3\, e^{-\frac{1}{2}x_3^2 + (s+2x_1)x_3}\, S_b\left[\frac{ia}{2} - x_3\right]$$





$-\frac{1}{2} - \frac{1}{4} = -\frac{3}{2}$

$a = 1$



$(Q,q)_n = (1-Q)(1-Qq)\cdots(1-Qq^{n-1})$

$Q = q^{i}$, i

$N_{\mu\nu}(Q, \{t, q\}) = N_{\nu^T\mu^T}(Q\frac{t}{q}, \{q, t\})$

$N_{\mu\nu}(Q, \{\frac{1}{t}, \frac{1}{q}\}) = N_{\nu^T\mu^T}(Q\frac{q}{t}, \{\frac{1}{q}, \frac{1}{t}\})$ ✓

$N_{\mu\nu}(\sqrt{\frac{t}{q}}\, Q^{-1}, \{\frac{1}{t}, \frac{1}{q}\})$

$\underset{(\checkmark)}{=} (-Q)^{-|\mu|-|\nu|}\, t^{-\frac{\|\mu^T\|^2}{2}+\frac{\|\nu^T\|^2}{2}}\, q^{\frac{\|\mu\|^2}{2}-\frac{\|\nu\|^2}{2}}$

✓ $N_{\nu\mu}(\sqrt{\frac{t}{q}}\, Q, \{\frac{1}{t}, \frac{1}{q}\})$

checked

$\cancel{N_{\mu\nu}(\tilde{Q}\sqrt{\frac{t}{q}}, \{\frac{1}{t}}$

$N_{\mu\nu}(\tilde{Q}\sqrt{\frac{t}{q}}, \{\frac{1}{t}, \frac{1}{q}\}) \underset{(\checkmark)}{=} N_{\nu^T\mu^T}(\tilde{Q}\sqrt{\frac{q}{t}}, \{\frac{1}{q}, \frac{1}{t}\})$

$Q = \tilde{Q}\sqrt{\frac{t}{q}}$,

$\tilde{Q} = Q\sqrt{\frac{q}{t}}$

$Q_1 = Q = \tilde{Q}\sqrt{\frac{t}{q}}$,

$Q_2 = Q\frac{q}{t} = \tilde{Q}\sqrt{\frac{q}{t}}$

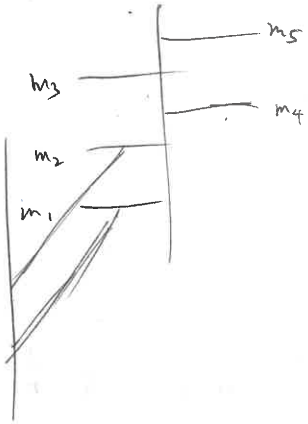
$(1 - q^{\frac{3}{2}} t\, \tilde{Q}\sqrt{\frac{t}{q}})$ $\quad (1 - q^{\frac{3}{2}} t^{\frac{3}{2}} \tilde{Q})$

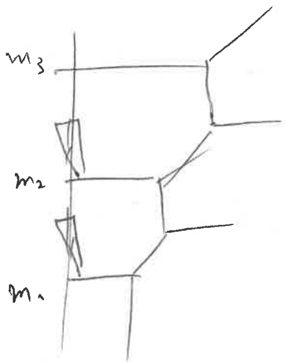
$\cancel{(1 - q^2 t\, \tilde{Q}\sqrt{\frac{t}{q}}}$

$\cancel{1 - q^2 t}$ $\quad 1 - q t^2 \cdot \tilde{Q}\frac{\sqrt{q}}{\sqrt{t}}$

$1 - q^{\frac{3}{2}} t^{\frac{3}{2}} Q$

* (q, t)-brane for $U(N) + N_f F + N_a \tilde{F}$
* decouple F or $\tilde{F}$ for interesting webs
* eff CS level for $U(1)$ brane webs and its flips
* extra open strings for $U(N)$
* abelianization for $U(N)$





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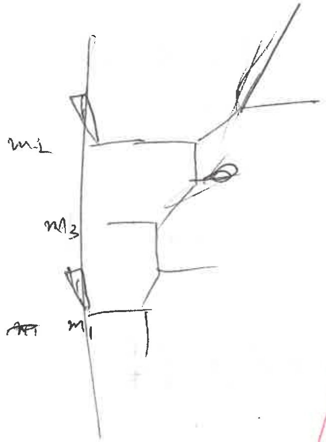
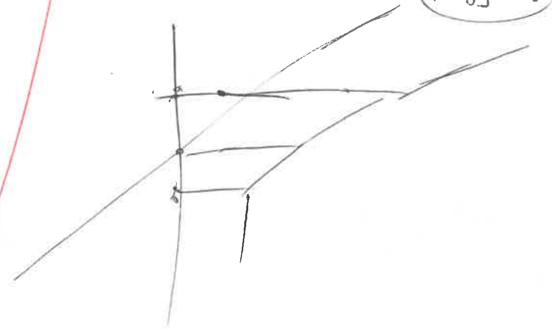
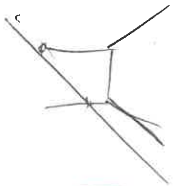
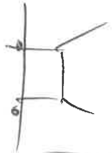




$C_5^2 = \frac{5 \times 4}{2} = 10$

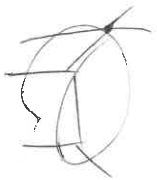
$2 \cdot C_4^2$

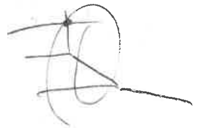




$k \overset{?}{=} 1$

$k_{eff} \overset{?}{=} 2$





$a_2 \to a_2^{-1} \; f_{a_1}$

$a_1 \to a_1^{-1} \; f_a$

$\binom{5}{a} * f$

Jun-11

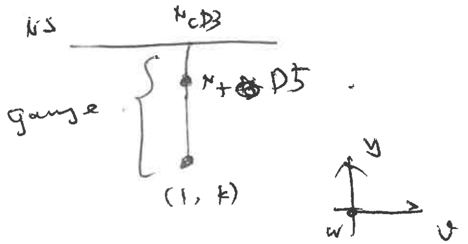


Fig 2b

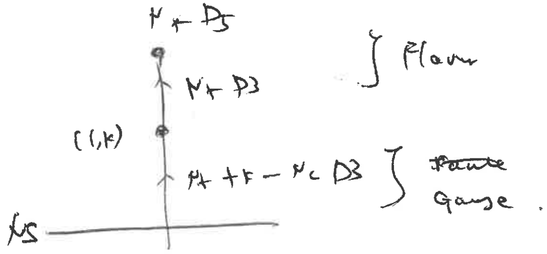
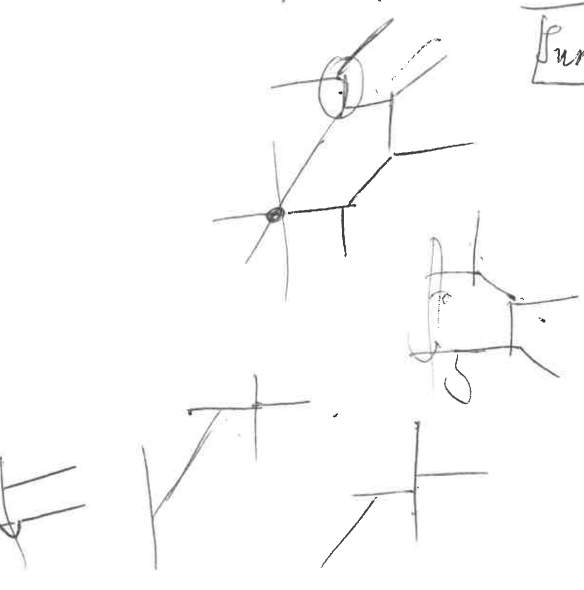
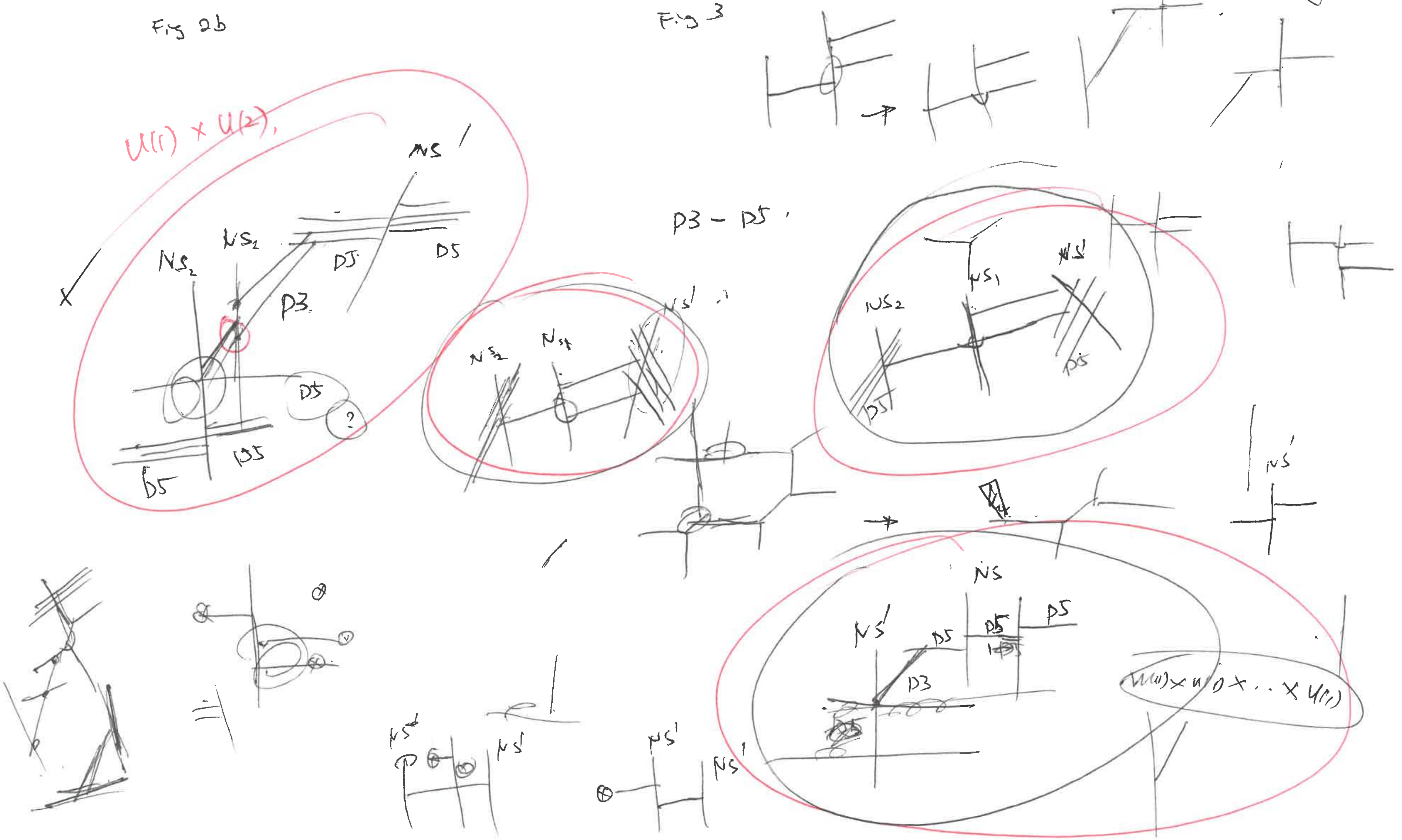


Fig 3



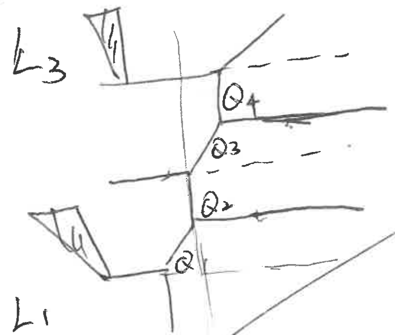


July ≏ 17

$$(Q,q)_k = \frac{(Q,q)_\infty}{(Qq^k,q)_\infty} = PE\left[\frac{Q(q^k-1)}{1-q}\right]$$

$$= \exp\left(\sum_{n=1}^{\infty}\frac{1}{n}\frac{Q^n q^{kn}}{1-q^n}\right)\exp\left(-\sum_{n=1}^{\infty}\frac{1}{n}\frac{Q^n}{1-q^n}\right)$$

$$= \exp\left[\sum_{n=1}^{\infty}\frac{1}{n}\frac{Q^n(q^{kn}-1)}{1-q^n}\right]$$



$$\left(\frac{\sqrt{t}\,Q\nu_1}{\sqrt{q}\,Q_1}\right)^{n_1}\left(\frac{\sqrt{q}\,Q\nu_3}{\sqrt{t}\,Q_3}\right)^{n_3}$$

$$(-\sqrt{t})^{-2n_1 n_3}$$

$$(Q_1 Q_2 Q_3 \sqrt{\tfrac{t}{q}}, t)_{n_1}$$

$$(Q_1\sqrt{\tfrac{t}{q}}, t)_{n_1}\ (Q_4^{-1}\sqrt{\tfrac{t}{q}}, t)_{n_3}\ (Q_2^{-1}Q_3^{-1}Q_4^{-1}\sqrt{\tfrac{t}{q}}, t)_{n_3}$$

May 29

2021 Jul 28

Mar 6 – 2022

$L[Q_{m_3}, \gamma_3]$ $L\left[\frac{Q_{m_1}Q_1Q_2}{Q_{m_2}Q_{m_4}}, \gamma_5\right]$

$L[Q_{m_4}, \gamma_4]$

$L[Q_{v_3}, \nu_3]$

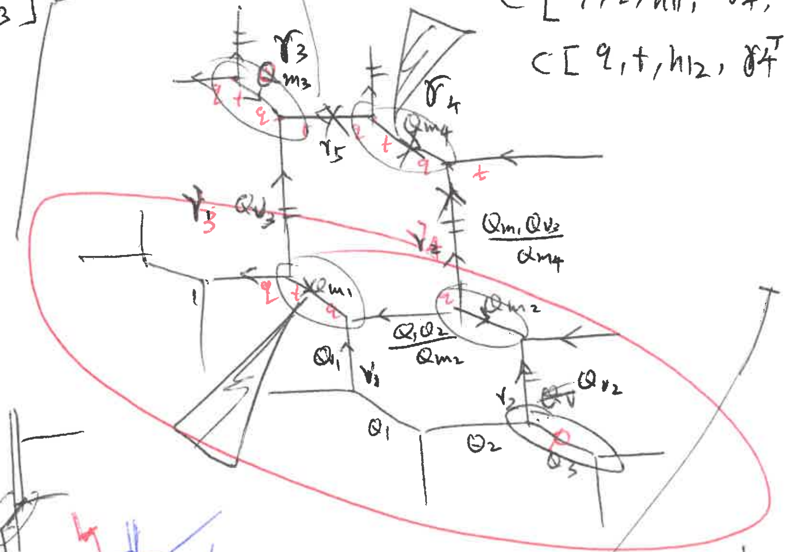
$L\left[\frac{Q_{m_1}Q_{v_3}}{Q_{m_4}}, \nu_4\right]$

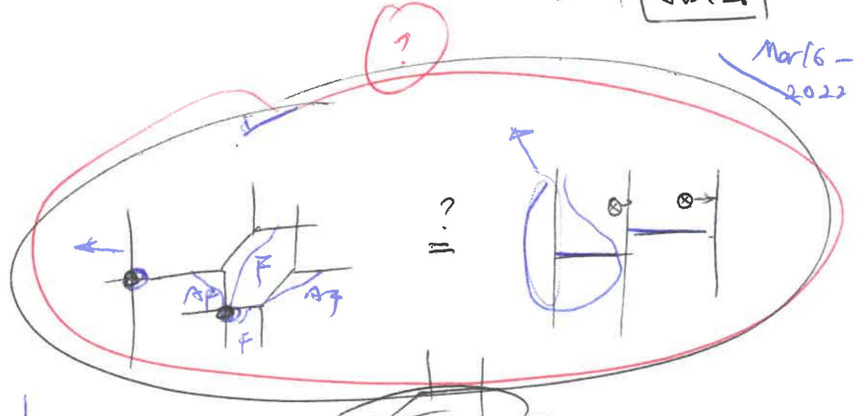
$C[t, q, h_9, \nu_3, \emptyset, \emptyset]$

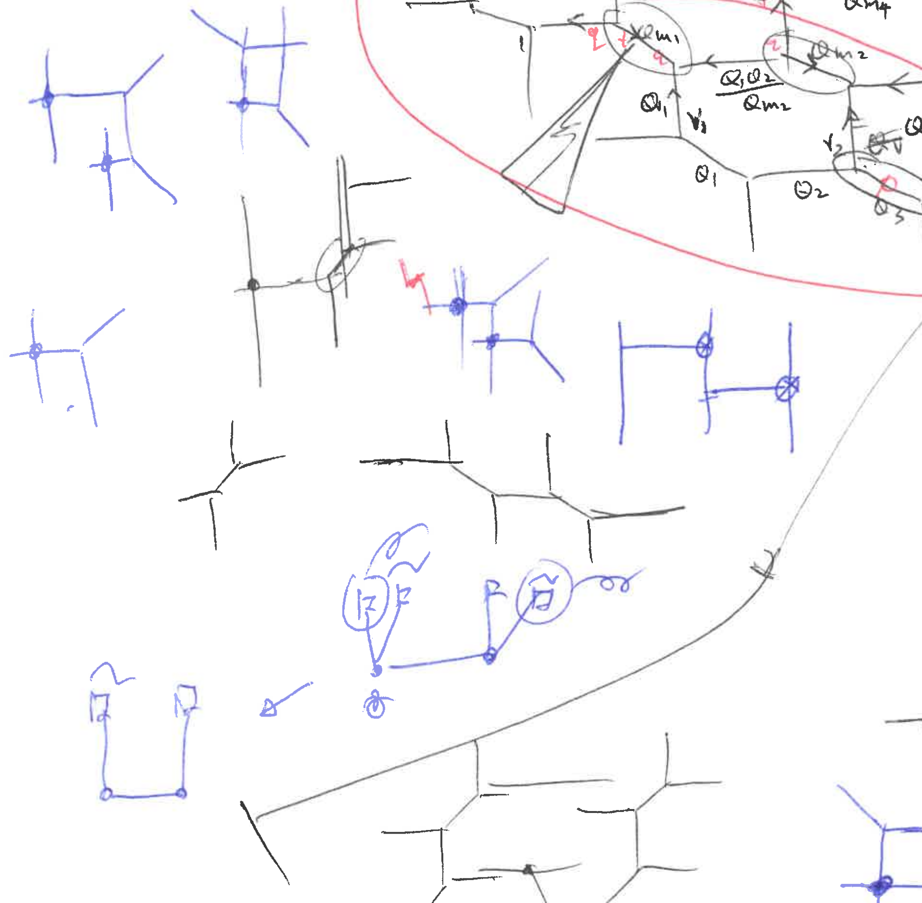
$C[q, t, h_{10}, \gamma_3^T, \gamma_5^T, \nu_3^T]$

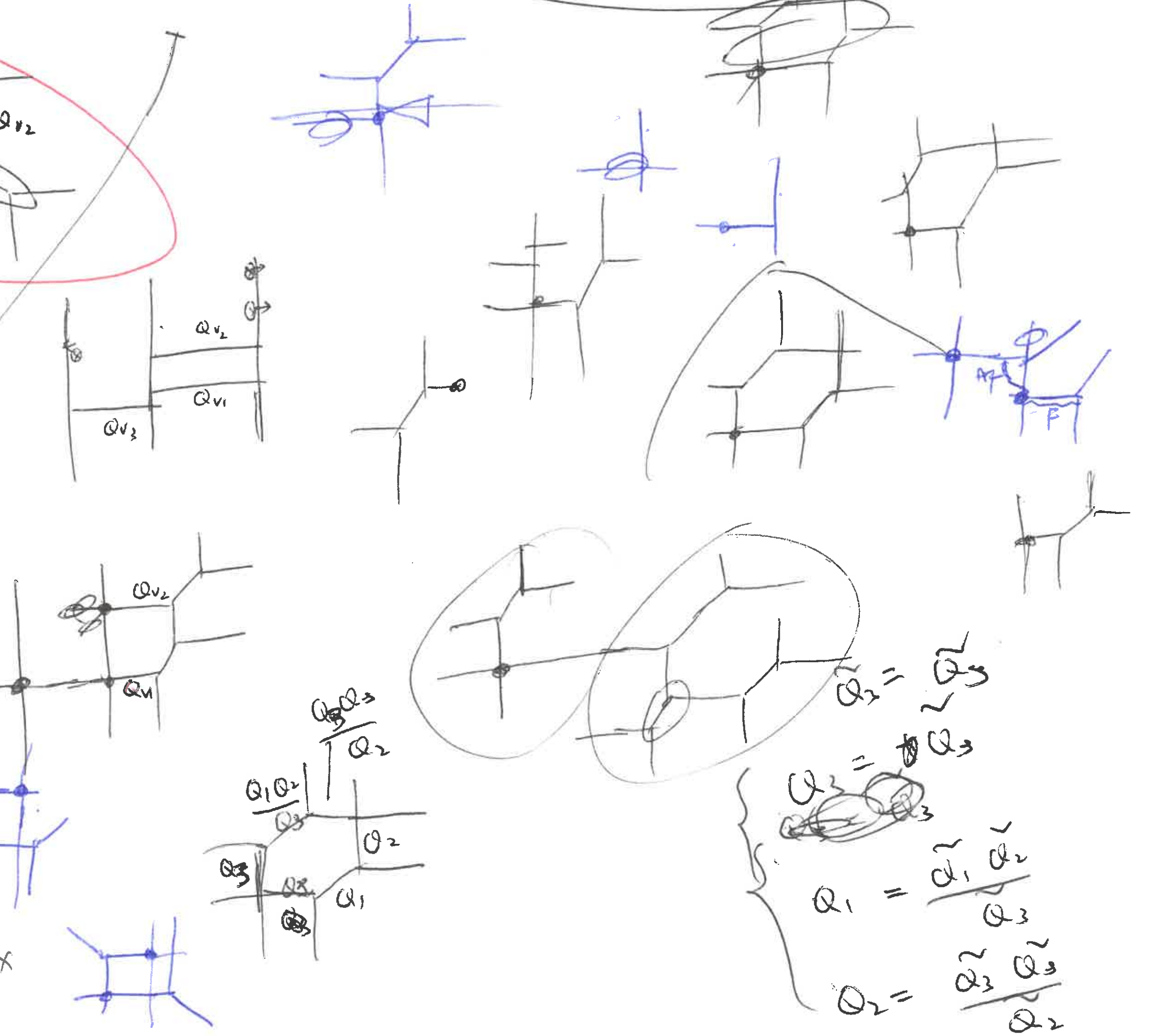
$C[t, q, h_{11}, \gamma_4, \gamma_5, \emptyset]$

$C[q, t, h_{12}, \gamma_4^T, \emptyset, \nu_4^T]$









$\tilde{Q}_2 = \tilde{Q}_3$

$Q_3 = \tilde{Q}_3$

$Q_1 = \frac{\tilde{Q}_1 \tilde{Q}_2}{\tilde{Q}_3}$

$Q_2 = \frac{\tilde{Q}_3 \tilde{Q}_3}{\tilde{Q}_2}$

$Q_3 Q_1 = \frac{Q_1 Q_2}{Q_3}$ ×

Jan 13

$W = \cdots \, \mathcal{S}\, \sigma$

$\mathcal{S} = \langle \phi \rangle$

$W = \phi\, \mathcal{S}'$

$W = \phi\, \mathcal{S}' +$

$W = \langle \phi \rangle\, \sigma$

$W = \langle \phi' \rangle\, \phi + \cdots\, \sigma$

$W = \phi' \phi + \cdots$

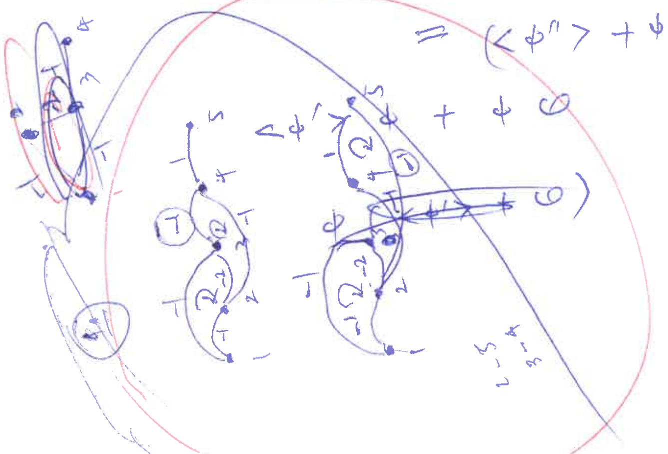
$W = \phi\, \sigma +$

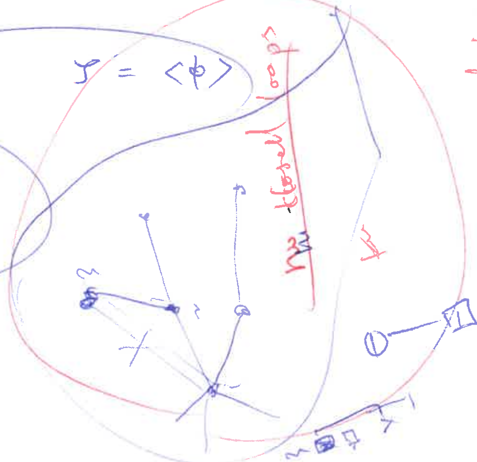
$W = \langle \phi'' \rangle \phi' + \phi' \phi + \phi\, \sigma$

$= \phi' (\phi + \langle \phi'' \rangle) + \phi\, \sigma$

$= (\langle \phi'' \rangle + \phi)\, \phi'$

$\phi\, (\phi' + \sigma)$





no closed loops

How to handle closed loops?

$Y = pd + X =$

$W = \Phi\, q\, \tilde{q}$

$A_i \wedge dA_i = (-1)^{\cdots}\, dA_j \wedge A_i = dA_j \wedge A_1$

$k_{ij} = k_{ji}$

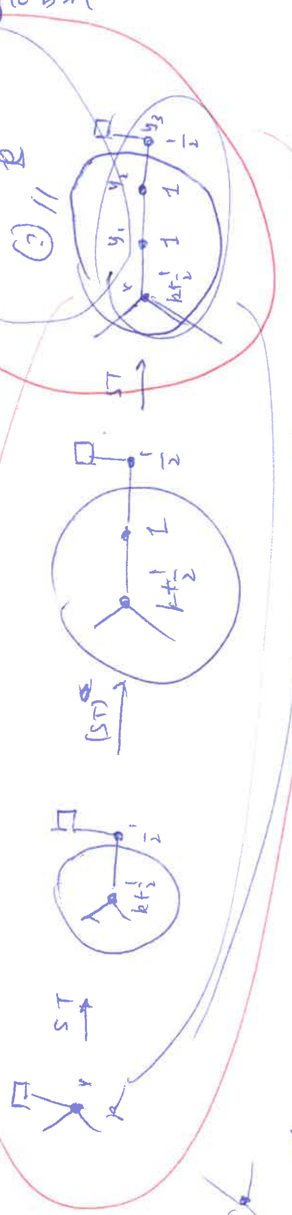
$e^{-2\pi i x_1 y_1}$

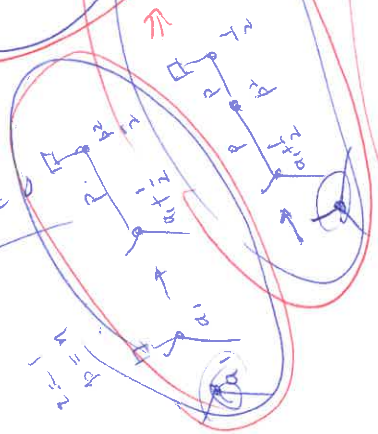
$e^{-2\pi i x_2 y}$

$x_1 y_1 + x_2 y_2 = 2 x_1 x_2$

$y =$

gauge global s.





$= \int dk_1 \int dk_2\, e^{-\pi i k_1 x_1^2}\, e^{-\pi i k_2 x_2^2}$

$e^{-2\pi i x_1 y_1}\, e^{-2\pi i x_1 x_2 y}$

$\oint dz\, e^{-2\pi i (y_2 - x_1) z_2}$

$-2\pi i\, k^2 y_3^2$

Basis

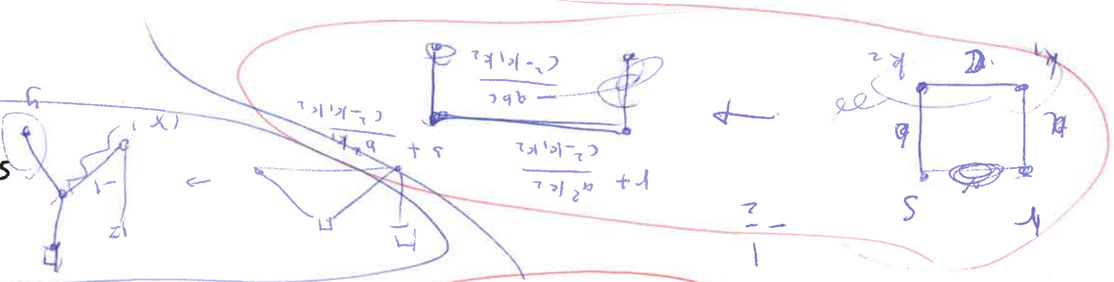
— Dynkin basis

$\downarrow$

Weight basis

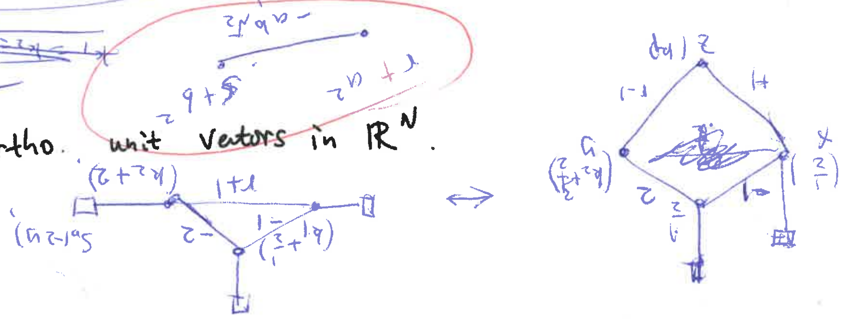
— α-basis

— Orthogonal basis



$SU(N)$ consider mutually-ortho. unit vectors in $\mathbb{R}^N$.

$$\vec{e}_i \cdot \vec{e}_j = \delta_{ij}$$



$$\vec{e}_1 = (1, 0, 0, \cdots)$$

root vectors of $SU(N)$ lie in the hyperplane $\vec{V} = \vec{e}_1 + \vec{e}_2 + \cdots \vec{e}_N$.

$SU(3)$, $\alpha_1 = e_1 - e_2$

$\alpha_2 = e_2 - e_3$

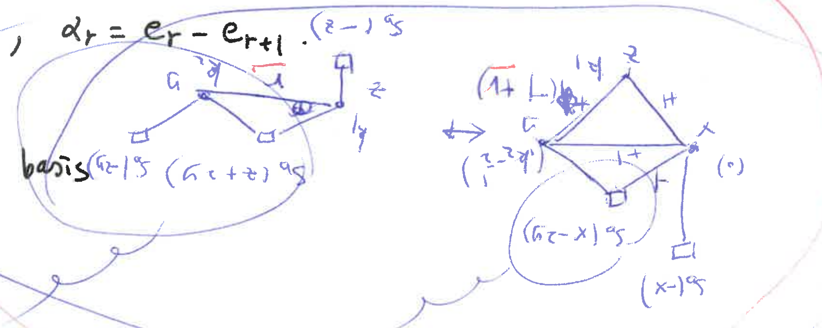
$(\alpha_1, \alpha_1) = 2$, $(\alpha_1, \alpha_2) = (e_1 - e_2, e_2 - e_3) = -1$.

$SU(N)$ o—o—o…o—o

$\alpha_1 = e_1 - e_2$, $\alpha_2 = e_2 - e_3$, …, $\alpha_r = e_r - e_{r+1}$

$SU(3)$

Dynkin basis vs Ortho. basis



$\alpha_1 = (2, -1) = e_1 - e_2$

$\alpha_2 = (-1, 2) = e_2 - e_3$

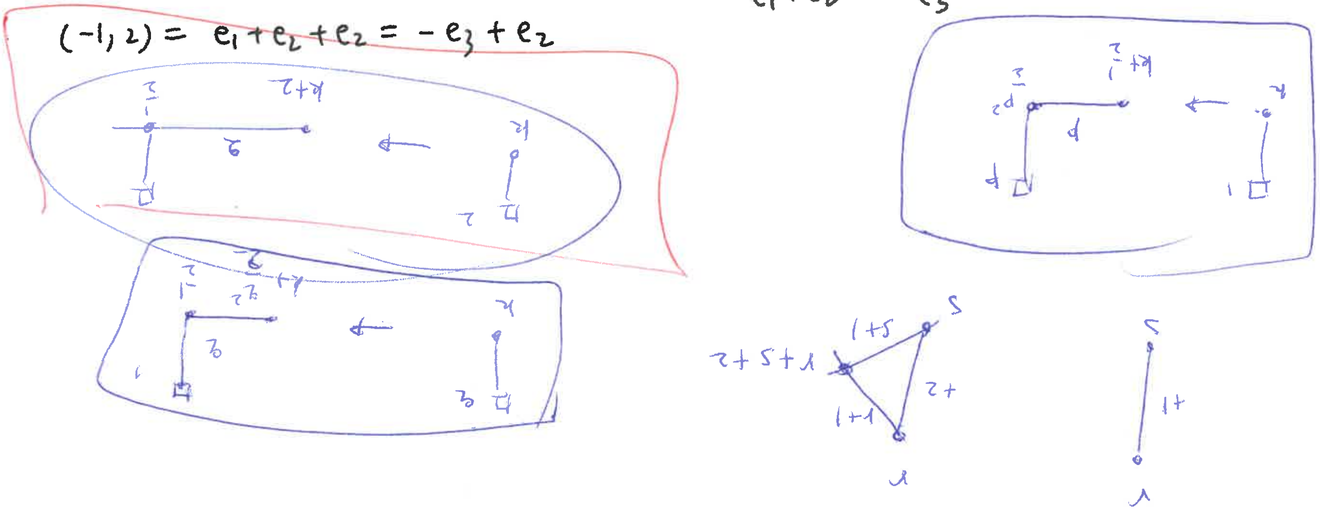
$(1, 0) = e_1$, $(0, 1) = e_1 + e_2$

$(2, -1) = e_1 - e_2$

$(-1, 2) = e_1 + e_2 + e_2 = -e_3 + e_2$

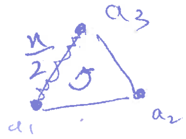
$\vec{V}_1 = \vec{e}_1 + \vec{e}_2 + \vec{e}_3 = 0$

$\vec{e}_1 + \vec{e}_2 = -\vec{e}_3$



Mar 02

Jan 13



if $a_2 = 0$ $\quad$? $=$

$$\begin{bmatrix} a_1 & \cancel{1} & * \\ 1 & a_2 & 1 \\ * & 1. & a_3 \end{bmatrix}$$

$$I = \oint dx_1 dx_2 dx_3 \, e^{-\pi i a_1 x_1^2 \cancel{+} \pi i a_2 x_2^2 - \pi i a_3 x_3^2} e^{-\pi i \cdot 2 x_1 x_2} \cdot e^{-\pi i 2 x_2 x_3} e^{-\pi i n x_1 x_3}$$

$\tilde{a}_1 \tilde{a}_2 = 1$, $\tilde{a}_2 \tilde{a}_3 = 1$ $\Rightarrow \begin{cases} \tilde{a}_2 = \frac{1}{\tilde{a}_1} \\ \tilde{a}_3 = \frac{1}{\tilde{a}_2} = \tilde{a}_1 \end{cases}$, $\quad a_1 \cdot a_3 = \cancel{a_1} = a_1^2 = \cancel{n}$?

$n \tilde{a}_1^2 = 2$, $\quad \tilde{a}_1 = \sqrt{\frac{2}{n}}$.

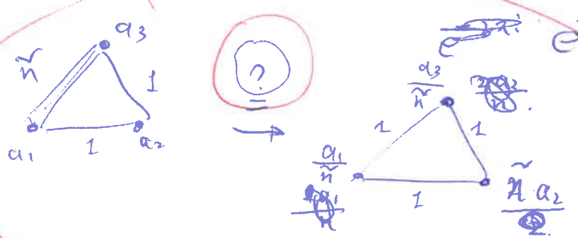
$\boxed{n \Rightarrow 2 \tilde{n}}$

$\tilde{a}_3 = \tilde{a}_1 = \sqrt{\frac{2}{n}}$, $\quad \tilde{a}_2 = \sqrt{\frac{n}{2}}$

$$I \Rightarrow \oint d\tilde{x}_1 d\tilde{x}_2 d\tilde{x}_3 \, \cancel{e^{-\pi i a_1 \tilde{a}_1^2 x_1^2 - \pi i a_2 \tilde{a}_2^2 x_2^2}}$$

$\begin{cases} x_1 \to \tilde{a}_1 \tilde{x}_1 \\ x_2 \to \tilde{a}_2 \tilde{x}_2 \\ x_3 \to \tilde{a}_3 \tilde{x}_3 \end{cases}$

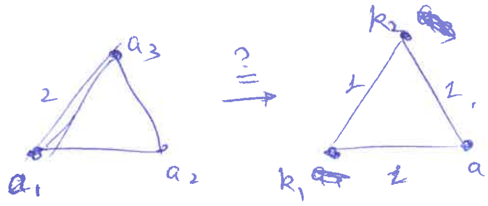
$$= e^{-\pi i \frac{a_1 \cdot 2}{n} \tilde{x}_1^2} e^{-\pi i \frac{a_2 n}{2} \tilde{x}_2^2} e^{-\pi i \frac{2 a_3}{n} \tilde{x}_3^2} \cdot e^{-\pi i \cdot 2 \tilde{x}_1 \tilde{x}_2} e^{-\pi i \cdot 2 \tilde{x}_2 \tilde{x}_3} e^{-\pi i \cdot 2 \tilde{x}_1 x_3}$$

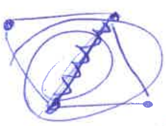


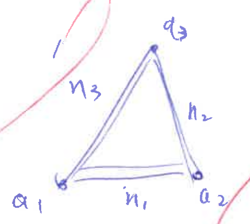
$\begin{cases} a_1 = \tilde{n} m_1 \\ a_3 = \tilde{n} m_2 \end{cases}$

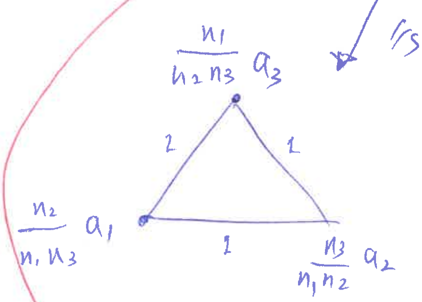
$\cancel{n \Rightarrow 2n}$,

ex $\quad \tilde{n} = \cancel{A}^2$, $\quad a_1 = 2\cancel{k_1}$, $\quad a_3 = 2k_2$









$$I = \oint dx_1 dx_2 dx_3 \, e^{-\pi i a_1 x_1^2 - 2\pi a_2 x_2^2 - \pi i a_3 x_3^2} e^{-\pi i 2 n_1 x_1 x_2} e^{-\pi i \cdot 2 n_2 x_2 x_3} e^{-\pi i \cdot 2 n_3 x_1 x_3}$$

$\{ x_i \to \tilde{a}_i \tilde{x}_i$, $\quad \begin{cases} n_1 \tilde{a}_1 \tilde{a}_2 = 1 \\ n_2 \tilde{a}_2 \tilde{a}_3 = 1 \\ n_3 \tilde{a}_1 \tilde{a}_3 = 1 \end{cases}$

$\begin{cases} \tilde{a}_2 = \frac{1}{n_1 \tilde{a}_1} \\ \tilde{a}_3 = \frac{1}{n_2 \tilde{a}_2} = \frac{n_1 \tilde{a}_1}{n_2} \\ \cancel{\frac{n_3 \tilde{a}_1^2 n_1}{n_2}} = 1 \end{cases}$ $\quad a_1$

$\frac{n_1^2}{n_2^2} \cdot \frac{\cancel{n_2}}{n_1 n_3}$

$\frac{a_1 n_3}{n_2}$

$\tilde{a}_1^2 = \frac{n_2}{n_1 n_3}$

$\frac{n_2}{n_1 n_3}(a_1 + \frac{1}{2})$ $\quad \cancel{\frac{n_2}{n_1 n_3} a_1} + \frac{1}{2}$ $\quad \frac{n_2}{n_1 n_3}$

HW $3 \times 3 = 6 + \bar{3}$

$SU(3) \supset SU(2) \times U(1)$

$6 = 3_{(\ )} + 2_{(\ )} + 1_{(\ )}$

$3a + 2b + c = 0$

$6_{(2,0)}$

$q^b (x + \frac{1}{x})$

G_2 

$$A_{ij} = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

$$A_{12} = \frac{2(\alpha_1, \alpha_2)}{(\alpha_2, \alpha_2)} = -3 \Rightarrow (\alpha_1, \alpha_2) = -\frac{3}{2} (\alpha_2, \alpha_2)$$

$$\|\alpha_1\|^2 = 3\|\alpha_2\|^2$$

α_1 α_2

$B_2 = \begin{pmatrix} 2 & -2 \\ -1 & 2 \end{pmatrix}$  α_1

C_2  $\begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix}$

HW: "G_2" exceptional  α_1 α_2

$(1,0) \to$ fund $\to 7$

$(0,1) \to$ adj $\to 14$

$\boxed{1,0}$

$\vdots$

$\alpha_1 = (2, -1)$

$\alpha_2 = (-3, 2)$

$G_2 \supset SU(3)$

f : $7 = 3 + \bar{3} + 1$

adj : $14 = 8 + \cdots$

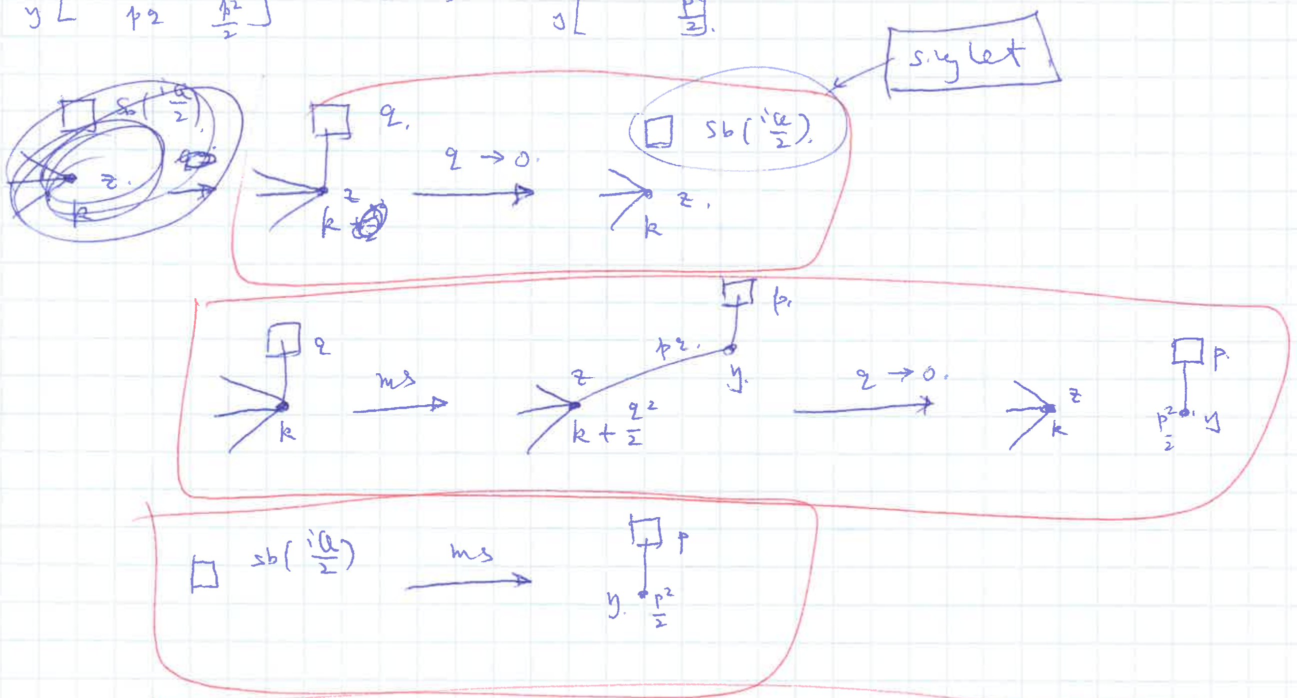
Feb 4 Feb 2

$$S_b\left(\frac{iQ}{2} - qz\right) \overset{ms}{=} p\int dy\, e^{-\frac{i\pi}{2}p^2y^2 - 2\pi i\left(\frac{iQ}{4} - qz\right)py}\, S_b\left(\frac{iQ}{2} - py\right)$$

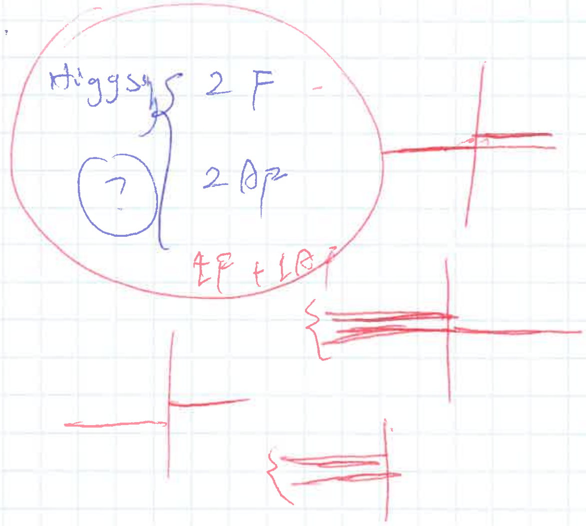
If $q = 0$,

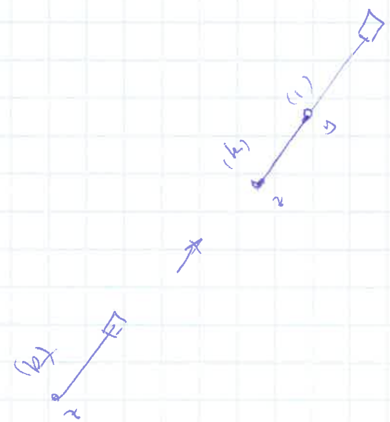
$$\boxed{S_b\left(\frac{iQ}{2}\right) = p\int dy\, e^{-\frac{i\pi}{2}p^2y^2 + \frac{2\pi Q}{4}py}\, S_b\left(\frac{iQ}{2} - py\right)}$$

$$\begin{array}{c|cc} & z & y \\ \hline z & k+\frac{q^2}{2} & pq \\ y & pq & \frac{p^2}{2} \end{array} \xrightarrow{q\to 0} \begin{array}{c|cc} & z & y \\ \hline z & k & \\ y & & \frac{p^2}{2} \end{array}$$



Any gauge node $\to k$ can be attached w/ a singlet by turning on the charge q





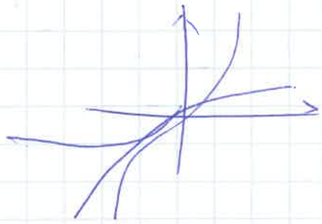
$$\sqrt{\frac{3M}{M^2}}\,\phi = \text{arc tan}$$

$$\text{arc tan}\left(\frac{\sqrt{3m}\,\phi}{M}\right) = -\frac{M^2}{3\,H}\,\frac{\sqrt{3m}}{\cancel{\ }}$$

$$= -\frac{Mm}{\sqrt{3}\,H}\,t\,.$$

$\tan\phi =$

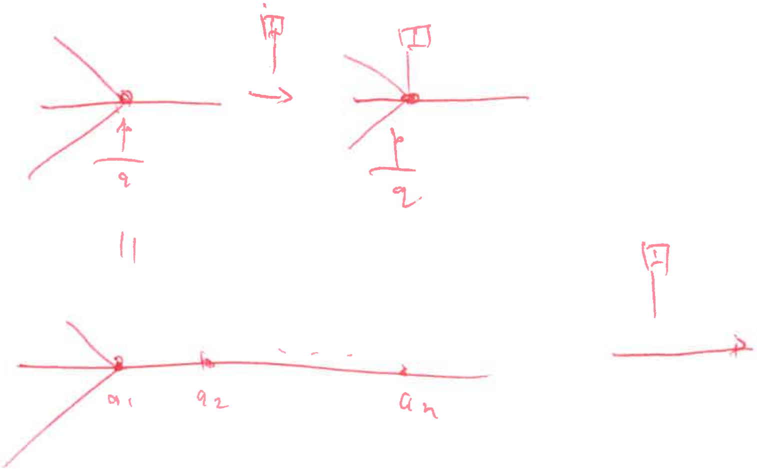
$$\frac{M}{M^2\sqrt{3m}}\int \frac{\frac{\sqrt{3m}\,\phi}{M}}{1+\left(\frac{\sqrt{3m}\,\phi'}{M}\right)^2}$$

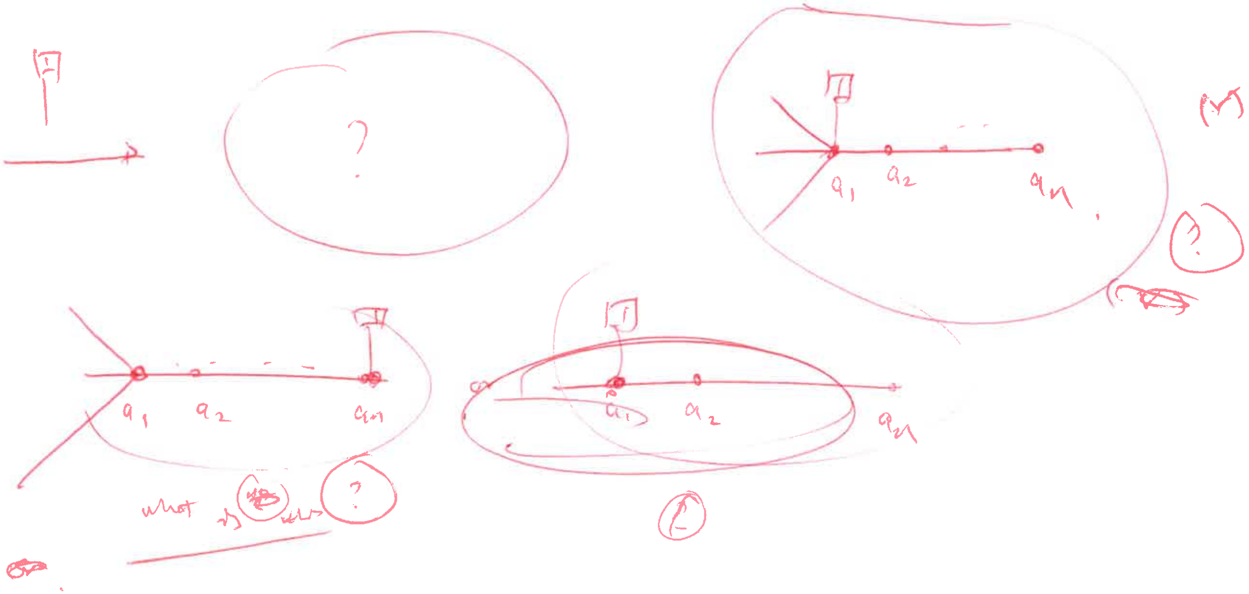


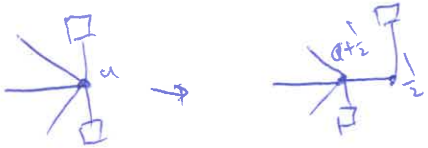
arc tan

Jan 21

? How to introduce matters after expansion of ~~fractional~~ the gauge node w/ fractional cs level ?







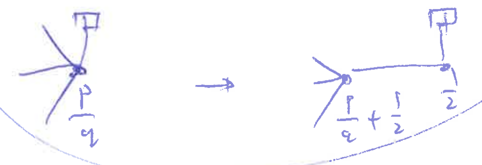
Jan 21

$$a_4 + \frac{2-a_2}{a_2a_3-1} = \frac{a_2}{a_2a_3-1}$$

$$a_4 - \cfrac{a_2}{a_2a_3-1}$$

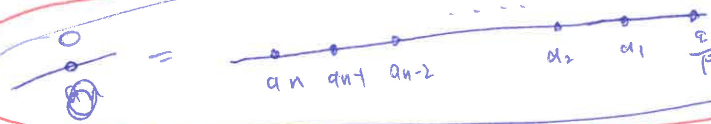
$$\frac{1}{a_4 + a_2(1-a_3a_4)}$$

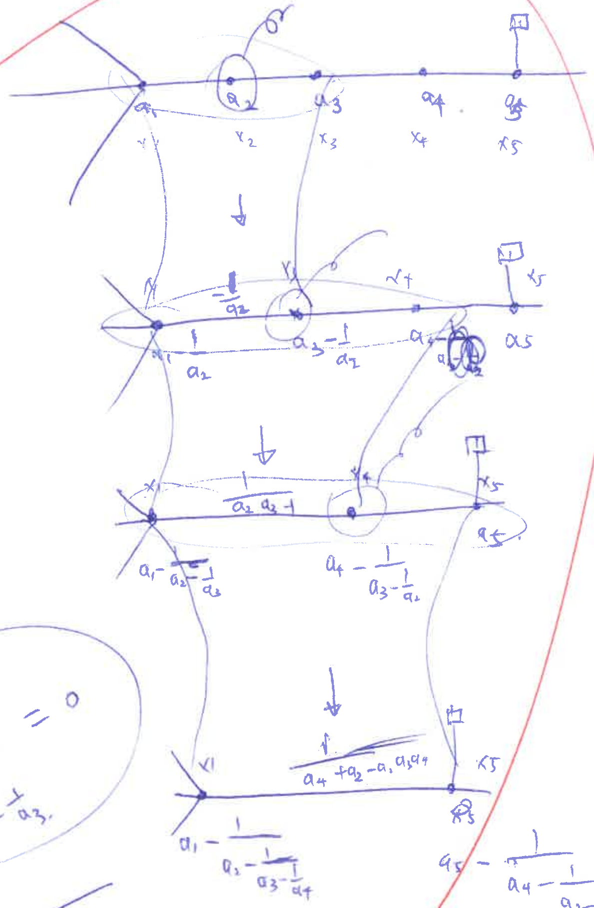
$$\frac{1}{a_4 + a_2 - a_2a_3a_4}.$$



$$\frac{p}{q} = a_1 - \cfrac{1}{a_2 - \cfrac{1}{a_3 - \cdots \cfrac{1}{a_{n-1} - \cfrac{1}{a_n}}}}.$$

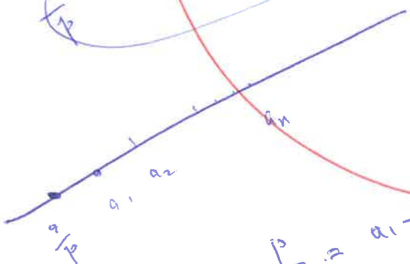
$$0 = a_n - \cfrac{1}{a_{n-1} - \cfrac{1}{a_{n-2} - \cdots \cfrac{1}{a_1 - \frac{q}{p}}}}$$





$$\frac{q}{p} = \cfrac{1}{a_1 - \cfrac{1}{a_2 - \frac{1}{a_3}}} = 0$$

$$a_5 - \cfrac{1}{a_4 - \cfrac{1}{a_3 - \frac{1}{a_2}}}$$



$$\frac{p}{q} = a_1 - \cfrac{1}{a_2 - \frac{1}{a_3}}$$

$$a_1 - \frac{1}{a_2}$$

$$\cfrac{1}{a_1 - \frac{p}{q}} = a_2 - \frac{1}{a_3}$$

$$a_2 - \cfrac{1}{a_1 - \frac{p}{q}} = \frac{1}{a_3}$$

$$a_3 = \cfrac{1}{a_2 - \cfrac{1}{a_1 - \frac{q}{p}}}$$

Jan 26

chiral $(z q; q)_\infty$

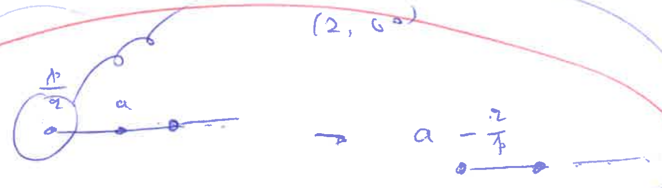
anti-chiral $(z^{-1} q, q)_\infty$

$(z, q; q)_\infty = PE\left[- \frac{z q}{1-q} \right]$

$\frac{1}{(q z^{-1}, q)_\infty} = PE\left[\frac{q z^{-1}}{1-q} \right]$

$(z q; q)(q z^{-1}, q)_\infty = PE\left[\frac{q(z^{-1} - z)}{1-q} \right]$

$\frac{q}{z} - q z = q (z$



$(2, 6^\infty)$

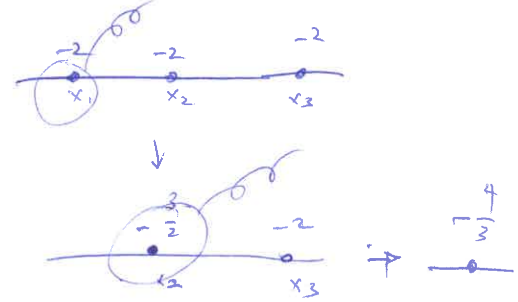
$a - \frac{q}{p}$

$\frac{a p - q}{p} = a - $

$(-)L(r, p) = L(r, r-p)$

$= \cancel{(-) L(r, } p^{-1}$

$= L(p, r)$ $a_1 - \frac{q}{p}$



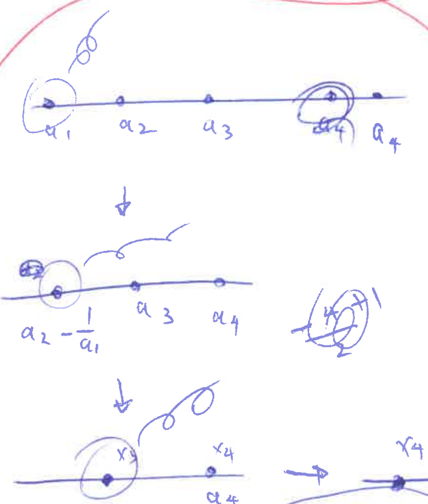
$\boxed{L(p, q)}$ $\frac{p}{q} = a_1 - \cfrac{1}{a_2 - \cfrac{1}{a_3 - \cfrac{1}{a_4}}}$

$a_1 - \frac{p}{q} = \cfrac{1}{a_2 - \cfrac{1}{a_3 - \cfrac{1}{a_4}}}$

$\frac{1}{a_1 - \frac{p}{q}} = a_2 - \cfrac{1}{a_3 - \cfrac{1}{a_4}}$

$-\frac{3}{2} = (-2) - \frac{1}{(-2)}$

$-\frac{3}{2}$ = -2 -2



a_1 a_2 a_3 a_4

$a_2 - \frac{1}{a_1}$ a_3 a_4

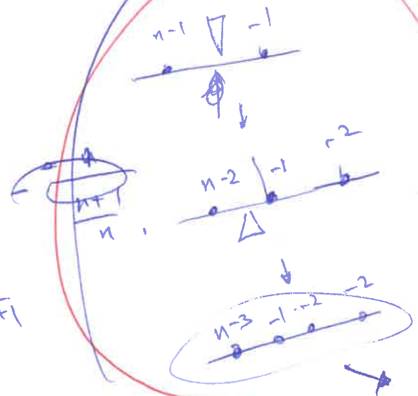
$a_3 - \frac{1}{a_2 - \frac{1}{a_1}}$ x_4 a_4

$\frac{p}{q} = a_4 - \cfrac{1}{a_3 - \cfrac{1}{a_2 - \frac{1}{a_1}}}$

$-\frac{3}{2} = -2 + \frac{1}{2} = (-2) - \frac{1}{(-2)}$

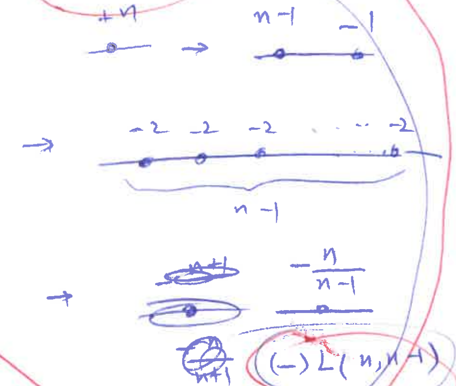
$- \frac{n-1+2}{n+1}$

$= -1 + \frac{2}{n+1}$



$L(n, 1)$

A_{n-1} :



$L(n,1) = (-) L(n, n-1)$

Contents

Jan 26.

$\oplus F$, $(q z^{-1}, q)_\infty$

$\oplus F$ $(qz, q)_\infty = (z, q^{-1})^{-1}_\infty$

$= \theta(z, q)$

$(qy, q)_\infty \xrightarrow{\hbar \to 0} e^{\frac{1}{2\pi i \hbar}\left[\mathrm{Li}_2(y) - \frac{1}{2}\log(1-y)\right]}$

$\theta(z, q) := (qz, q)_\infty \left(\frac{q}{z}, q\right)_\infty$

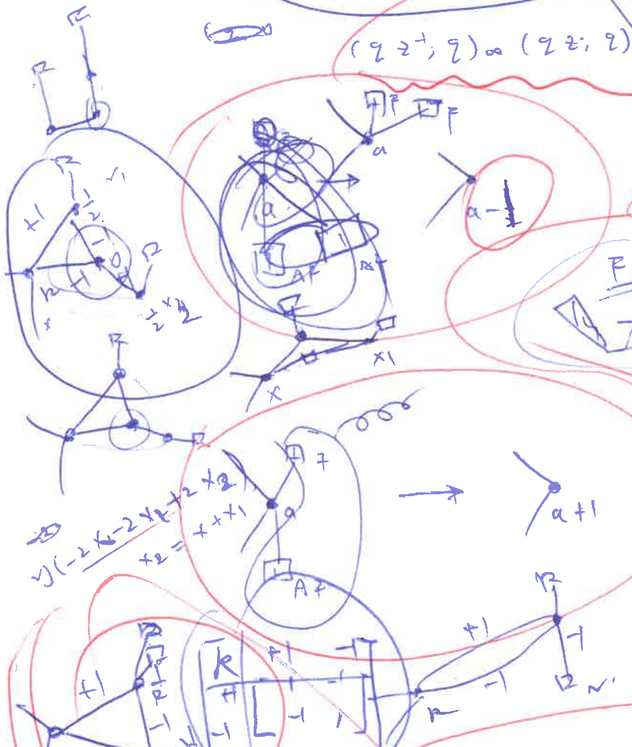
$\begin{bmatrix} k + \frac{q^2}{2} & p\cdot q \\ p\cdot q & \frac{p^2}{2} \end{bmatrix}$

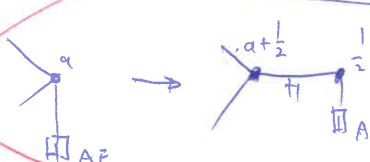
$(q z^{-1}; q)_\infty (q z; q)_\infty \to \exp\left(\underline{\mathrm{Li}_2(z) + \mathrm{Li}_2(z^{-1})} - \frac{1}{2}\log(1-z) - \frac{1}{2}\log(1-z^{-1})\right) \simeq \left(-\frac{1}{2}\right)[\log(-z)]^2$

$= -\frac{\pi^2}{6} - \frac{1}{2}(\log(-z))^2$

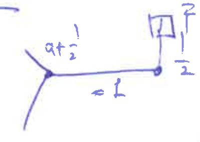
$q = -1, \quad p = -1$

$-\frac{1}{2}\log(1-z) - \frac{1}{2}\left(\log(z-1) - \log z\right)$





or



decouple $F \oplus AF$

$(x, q)_\infty = \frac{1}{(xq^n, q)_{-n}}$

$\mathrm{Li}_2(y) + \mathrm{Li}_2\left(\frac{1}{\frac{y-1}{y}}\right) + \mathrm{Li}_2\left(\frac{y}{y-1}\right)$

$\log\left(e^{-\pi i b^2 + \pi i}\right)$

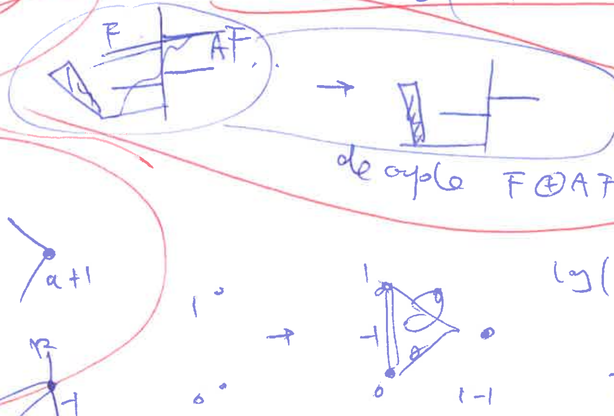
$\frac{-\pi i b^2 + \pi i}{-2\pi b} = \frac{b}{2}$

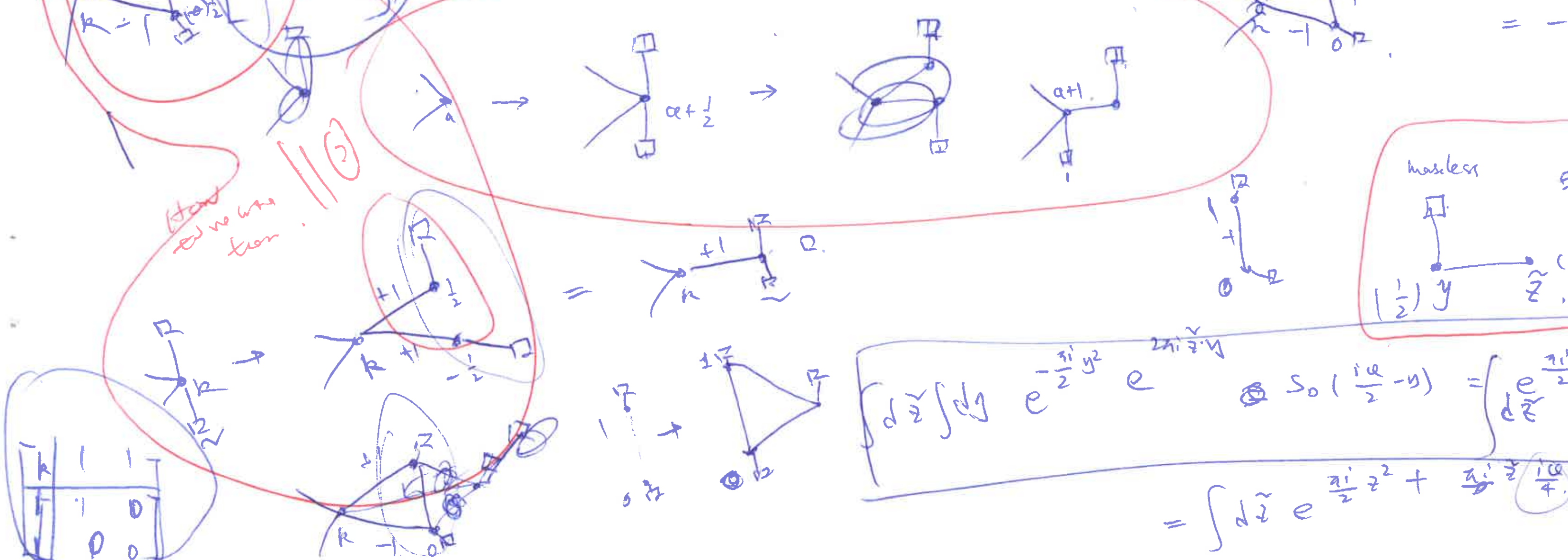
$\exp\left[\frac{1}{2}x_1^2 + 2x_m x_1 - 2x x_2 - \frac{x_2^2}{2} - 2x_1 x_2\right]$

$y = 1 - z$, $z = 1 - y$

$1 - \frac{1}{1-y} = \frac{-y}{1-y}$

$\mathrm{Li}_2(y) + \mathrm{Li}_2\left(\frac{y}{y-1}\right) = -\frac{1}{2}(\log(1-y))^2$





How do we turn...

massless

$\equiv_{ms}$

$\tilde z = \frac{ia}{4} - z$,

$\int d\tilde z \int dy\, e^{-\frac{\pi i}{2}y^2} e^{2\pi i \tilde z y} S_0\left(\frac{ia}{2} - y\right) = \int d\tilde z\, e^{\frac{\pi i}{2}\left(\tilde z + \frac{ia}{4}\right)^2} S_0\left(\frac{ia}{2} + \tilde z - \frac{ia}{4}\right)$

mass

$= \int d\tilde z\, e^{\frac{\pi i}{2}\tilde z^2 + \pi i \tilde z \frac{ia}{4} - \frac{\pi i a^2}{2\times 16}} S_0$

$\begin{bmatrix} k & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

USTC-ICTS-14-03
Bonn-TH-2014-02

Quantum geometry of del Pezzo surfaces in the Nekrasov-Shatashvili limit

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We use mirror symmetry, quantum geometry and modularity properties of elliptic curves to calculate the refined free energies in the Nekrasov-Shatashvili limit on non-compact toric Calabi-Yau manifolds, based on del Pezzo surfaces. Quantum geometry here is to be understood as a quantum deformed version of rigid special geometry, which has its origin in the quantum mechanical behaviour of branes in the topological string B-model. We will argue that, in the Seiberg-Witten picture, only the Coulomb parameters lead to quantum corrections, while the mass parameters remain uncorrected. In certain cases we will also compute the expansion of the free energies at the orbifold point and the conifold locus. We will compute the quantum corrections order by order on $\hbar$, by deriving second order differential operators, which act on the classical periods.

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Mar ~~23~~ Mar 26

$$(-\sqrt{q})^{-d_1^2-2d_1d_2}\ (\sqrt{q})^{-d_1}$$

$$\sum_{d_1=1}^{\infty}(-\sqrt{q})^{2nd_1+d_1^2}\left(\frac{\beta_i}{\sqrt{q}}\right)^{d_1}\ \frac{1}{(q,q)_{d_1}}\ \sum_{d_2=1}^{\infty}(-\sqrt{q})^{2d_1d_2+d_2^2}\frac{(\sqrt{q})^{d_2}}{(q,q)_{d_2}}$$

$$e^{-kx^2}S_0(x)$$

$$= \sum \sum_{d_1=1}^{\infty}(-\sqrt{q})^{2nd_1+d_1^2+2d_1d_2}\left(\frac{\beta_i}{\sqrt{q}}\right)^{d_1}\sum_{d_2=1}^{\infty}(-\sqrt{q})^{d_2^2}\frac{(\sqrt{-q})^{d_2}}{(q,q)_{d_2}}$$

$$J(x^2, xy^{-1}, q)$$

$$(-\sqrt{q})^{d_2}\ \left(\frac{q^n\beta_i}{\sqrt{q}}\right)^{d_1}$$

$$\sum_{d_1=1}^{\infty}(-\sqrt{q})^{d_1^2}\left[\left(\frac{\beta_i}{\sqrt{q}}\right)(-\sqrt{q})^{2(n+d_2)}\right]^{d_1}\qquad J(xy, x^{-1}y;q)$$

$$q^{n+d_2}$$

$$= \int e^{x^2+sx}\,dx$$

What is this function?

$$\sum_{k=1}^{\infty}(-\sqrt{q})^{n^2}x^n$$

$$(-\sqrt{q})^{-n^2}\left(\frac{q}{q^n\beta_i}\right)^{d_1}$$

$$J(xy^{-1}, x^2; q) = (qx^2;q)_\infty\sum_{n=0}^{\infty}\frac{(xy^{-1})^n}{(q^{-1},q^{-1})_n\,(qx^2;q)_n}$$

$$J(x^2, xy^{-1}; q) = (qxy^{-1};q)_\infty\sum_{n=0}^{\infty}\frac{(x^2)^n}{(q^{-1},q^{-1})_n\,(qxy^{-1};q)_n}$$

$$q^n = z_i$$

$$\theta\left(-\frac{\beta_i}{\sqrt{q}}q^{n+d_2};q\right)$$

$$\int\frac{ds_i}{s_i}\ \frac{\theta(z^2)\ \theta(s_i)}{\theta(z^2s_i)\ \theta(s_i)}$$

$$(\beta_i q^n, q)_\infty = \sum_{d=0}^{\infty}(-\sqrt{q})^{2nd+d^2}\frac{\left(\frac{\beta_i}{\sqrt{q}}\right)^d}{(q,q)_d},$$

$$\tilde{\beta}_i = q$$

$$(-x^{-1})^n$$

$$(-\tfrac{1}{x})^n$$

$$\frac{(q,q)_\infty}{(z,q)_n} = \frac{(q\,q^n,q)_\infty}{\cancel{(z,z)_\infty}}$$

$\boxed{\beta_i = q}$ mistake

$$(-1)^n + x^{-n}$$

$$= (-x)^{-n}$$

$$(\beta_i q^n, q)_\infty\,(q,q)_\infty\ \sum_{d=0}^{\infty}(-\sqrt{q})^{2nd+d^2}\left(\frac{\beta_i}{\sqrt{q}}\right)^{d}\ (\tilde{\beta}_i q^n, q)^D_\infty$$

F_2

$$= \sum_{d=0}^{\infty}(-\sqrt{q})^{-n^2-2nd}\left(\frac{q}{\beta_i}\right)^d(\tilde{\beta}_i q^n, q)^D_\infty$$

F_2

$$s^{-1} = q^n$$

$$k(k+2)+1 = 0$$

$$k^2+2k+1 \qquad (k+1)^2 = 0 \qquad k = -1$$

Mar 27

$(q^{-1}z^{-1}; q^{-1})_\infty$

$= (\cancel{q^{-1}z^{-1}}\, ; q)_\infty$

x^{-1}

$q^{\frac{1}{2}} \quad q\,Q^{1}$

$q^{\frac{1}{2}} \quad (q^{a} x)^{-1}$ y

$y = q\,x^{-1}$

$$\boxed{\frac{(x,q)_n}{(x,q)_\infty} = (x q^n, q)_\infty^{-1}}$$

$$(Q^{-1}, q)_n\, (Q; q^{-1})_\infty = \frac{(Q^{-1}, q)_n}{(Q^{-1}, q)_\infty} = (Q^{-1} q^n ; q)_\infty^{-1}$$

$$= (Q, q^{-1})_n\, (Q; q^{-1})_\infty\, (-\sqrt{q})^n (\sqrt{q}\,Q)^{-n}$$

$$\frac{(Q, q^{-1})_n}{(Q, q^{-1})_\infty} = (Q\, q^{-n}; q^{-1})_\infty^{-1}$$

$$= (Q q^{-n}, q^{-1})_\infty^{-1} \left[(Q, q^{-1})_\infty\right]^2 (-\sqrt{q})^n (\sqrt{q}\,Q)^{-n}$$

$$= (Q q\, q^{-n}, q)_\infty$$

$$\frac{dz^{-1}}{z^{-1}} = \frac{dz}{z^2} \cdot \frac{(-1)}{z^{a} z^{-1}} \quad dz$$

$(z^{-1})' = (-1)\, z^{-2}$

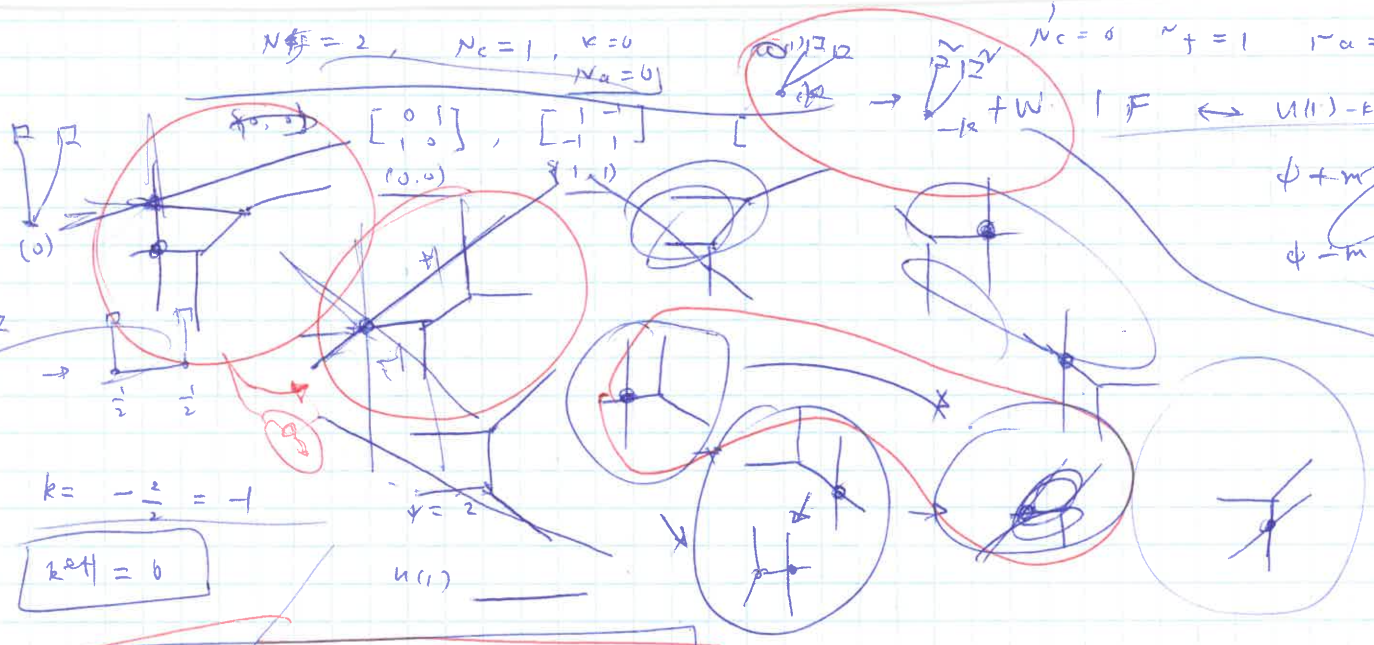
$\int \frac{dz}{z} \quad z \to z^{-1} \quad (-1)\int \frac{dz}{z}$

$N_f = 2$, $N_c = 1$, $k = 0$, $N_a = 0$

$N_c = 0$ $N_f = 1$ $N_a = 0$

~~Mar 26~~

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$



$1F \leftrightarrow U(1)_{-k} + \Phi + 1AF + W$

$\phi + m = 0$

$\phi - m = 0$

$N_c = 0$, $N_f = 0$

$N_a = 1$

$1AF \leftrightarrow U(1)_{-k} + 1F + W_{3d}$

$\zeta^{eff} = \zeta + \frac{\phi+m}{2} + \frac{\phi-m}{2}$

$\zeta^{eff} = \zeta + \phi$

$|x_1|^2 + |x_2|^2 = \boxed{\zeta + \phi} = 0$

$= \zeta^{eff}$

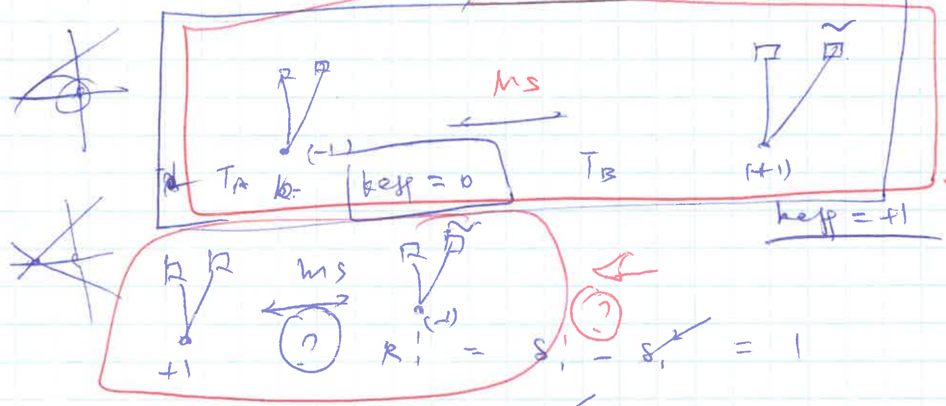
$\phi \pm m = 0$

$k = -\frac{2}{2} = -1$

$\boxed{k^{eff} = 0}$

$\psi = 2$

$U(1)$



MS

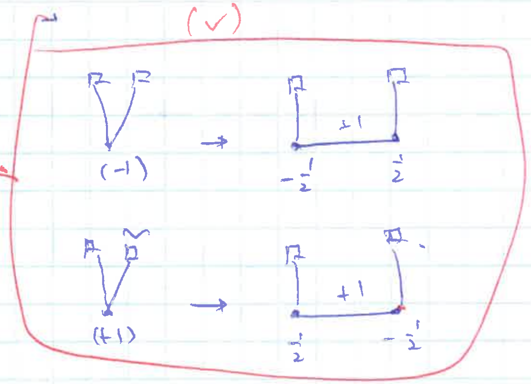
T_A k

$\boxed{k^{eff} = 0}$

T_B

(-1) $(+1)$

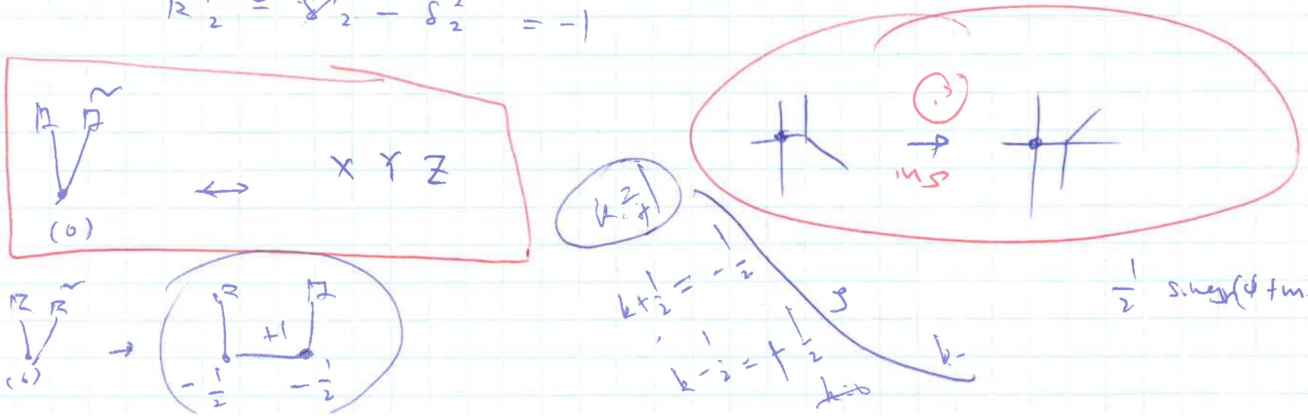
$k^{eff} = +1$



(v)

$k^1_1 = \delta^1_1 - \delta^2_1 = 1$

$k^1_2 = \delta^1_2 - \delta^2_2 = -1$



$\leftrightarrow$ XYZ

(0)

$k + \frac{1}{2} = -\frac{1}{2}$

$k - \frac{1}{2} = +\frac{1}{2}$

$\phi = \pm m$

$\frac{1}{2}\,\mathrm{sign}(\phi + m) + \frac{1}{2}\,\mathrm{sign}(\phi - m)$

Mar 22

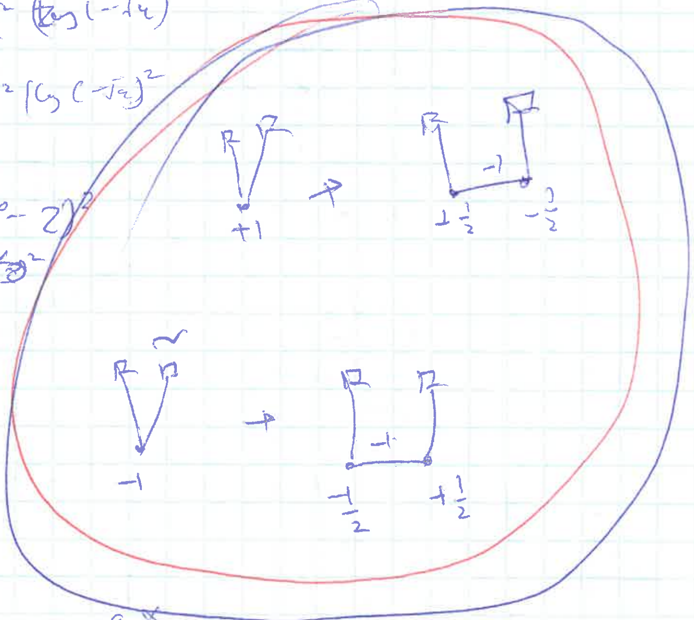
Apr 18

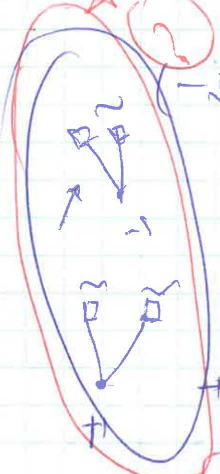
What is the difference between F and AF ?

$\frac{Z^2}{2} + \frac{\cancel{(\cdot\sqrt{2})}}{2}\frac{(a\log(-\sqrt{2}) + Z)^2}{2} - \frac{(b\log(-\sqrt{2}) + Z)^2}{2}$

$Z^2 + \cancel{Z^2} + 2a\, Z\log(-\sqrt{2}) + a^2(\log(-\sqrt{2}))^2$

$\cancel{-Z^2} - 2b\, Z\log(-\sqrt{2}) - b^2(\log(-\sqrt{2}))^2$





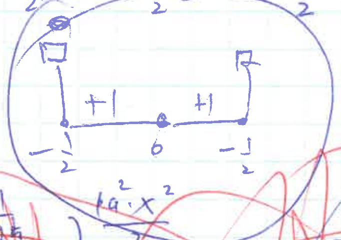
$(-\sqrt{2})^{b(a\cdot x)}$ $\quad a\log(-\sqrt{2})^a$

$\frac{2\cdot Y^2}{2} + \frac{2\cdot Z^2}{2} = \frac{(Y+Z)^2}{2} + \frac{(Y-Z)^2}{2}$

a

$\log Y + \log Z = \frac{2Y^2 + Z^2}{2}$

$-\frac{1}{2}x^2 - \frac{1}{2}s^2 + \frac{1}{2}(xs)^2$



$\left(\frac{x^2}{2} + \frac{s^2}{2} - \frac{(x+s)^2}{2}\right) \to xs$

$\frac{x^2+s^2-x^2-s^2+2xs}{2}$

$(e^{-\frac{1}{2}}) \quad \frac{a^2\cdot x^2}{2}$

$e^{a} =$

$1 + \sum_{n=1}^{\infty} q^{\frac{n^2}{2}} x^n + \sum_{n=1}^{\infty} q^{\frac{n^2}{2}} (x^{-1})^n$

$1 + F(x) + F(x^{-1}) = \theta(x)$

$1 + F(x) + F(x^{-1}) = \theta(x)$

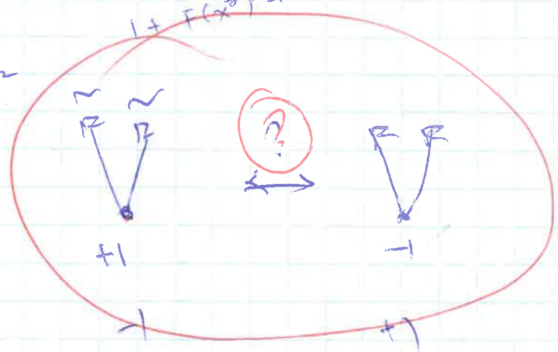
$1 + F(x^2) + F(x^{-1})$

$(a^2 - b^2)(\log(-\sqrt{2}))^2$

$Z^2 + 2(a-b)\log(-\sqrt{2})\, Z + (a^2-b^2)(\log(-\sqrt{2}))^2$

$\log z^2 = 2\log Z = 2Z$

$\frac{(Z + (a-b)\log(-\sqrt{2}))^2}{2}$



Apr 02

$(M f)(s) = \int_0^\infty \frac{dx}{x} \, x^{s} f(x)$

$e^{\frac{d\tilde{w}^{eff}}{d\log y_i}} = 1.$

$$\boxed{z_\alpha = 1 - y_\alpha \prod_j z_j^{Q_{j\alpha}}}$$

$c - 3 - 3 + 3 - 1$

$$\boxed{e^{\frac{d\,\mathrm{Li}_2(y)}{d\log y_\alpha}} = \frac{1}{1-y_\alpha}}$$

$(Q_+)\ (c-4)\times c.$

$1 - (1-z_\alpha)\, e^{z_\alpha \frac{\partial \tilde{w}^{eff}}{\partial z_\alpha}},$

$\frac{Q_{ij}}{2} \log z_\alpha z_\alpha^{\frac{Q_{ij}}{2}}$

$$\boxed{\frac{y_\alpha \cdot z_j^{Q_{j\alpha}}}{1-z_\alpha} = e^{z_\alpha \frac{\partial \tilde{w}^{eff}}{\partial z_\alpha}}}$$

$e^{s_i t} \to y_\alpha. \quad y_i \to z_j.$

$$\boxed{\tilde{w}^{eff} = \sum_{j=1}^{s,|Q|} \mathrm{Li}_2(z_j) + \sum_{i=1}^{c} \log y_\alpha \log(z_\alpha) + \sum_{i,j} \frac{Q_{ij}}{2} \log z_\alpha \log z_\alpha}$$

$z_\alpha \left[\frac{1}{1-z_\alpha} \cdot y_\alpha \cdot z_j^{Q_{j\alpha}} \right] = 1$

$z_\alpha \frac{y_\alpha \cdot z_j^{Q_{j\alpha}}}{1-z_\alpha}$

$$e^{\frac{d\tilde{w}^{eff}}{d\log z_j}} = \left(\frac{1}{1-z_j}\right) \cdot y_\alpha z_j^{Q_{j\alpha}} = 1,$$

$1 = e$

$y_\alpha z_j^{Q_{j\alpha}} = 1 - z_j.$

off-shell

$$\boxed{1 - y_\alpha z_j^{Q_{j\alpha}} = f(z_\alpha) = z_\alpha e^{z_\alpha \frac{\partial \tilde{w}^{eff}}{\partial z_\alpha}}}$$

$(1+z_\alpha)\, e^{\frac{d\tilde{w}^{eff}}{d\log z_\alpha}}$

$z_\alpha = 1 - y_\alpha z_j^{Q_{j\alpha}}.$

$(1-z_\alpha)\, e^{z_\alpha \frac{\tilde{w}^{eff}}{\partial z_\alpha}} = 0$

$z_\alpha = 0$ or

$1 - y_\alpha z_j^{Q_{j\alpha}} = z_\alpha$

$\frac{1}{1 - y_\alpha z_j^{Q_{j\alpha}}} = z_\alpha = z_\alpha e^{\frac{d\tilde{w}^{eff}}{d\log z_\alpha}},$

$\int \frac{dz_\alpha}{z_\alpha}$

$\prod_\alpha \left(\frac{e^{-z_\alpha \frac{\partial \tilde{w}^{r}}{\partial z_\alpha}}}{z_\alpha} \right) = \frac{e^{-\sum_\alpha z_\alpha \frac{\partial \tilde{w}^{eff}}{\partial z_\alpha}}}{\prod_\alpha z_\alpha}.$

$$\boxed{y = \int \frac{dz_\alpha}{z_\alpha} \frac{1}{1 - (1-z_\alpha)\, e^{z_\alpha \frac{\partial \tilde{w}^{eff}}{\partial z_\alpha}}}} = \frac{e^{-z_\alpha \frac{d\tilde{w}^{eff}}{\partial z_\alpha}},}{z_\alpha}$$

?

$$\boxed{S(y, M) = \int \frac{dz}{z} \frac{1}{\prod z_\alpha} \, S\left(e^{z_\alpha \frac{\partial \tilde{w}(z_\alpha)}{\partial z_\alpha}} - 1 \right)}$$ (*)

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$\boxed{L(p,q) \qquad \text{CS level } k = \frac{-p}{q}}$

$\boxed{L(p,1), \quad \text{CS level } k = -p}$

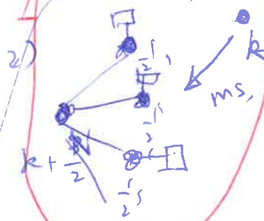
$$\frac{p}{q} = a_1 - \cfrac{1}{a_2 - \cfrac{1}{a_3 - \cfrac{1}{\cdots}}}$$

$\boxed{L(p,-q) = -L(p,q)}$

$$\frac{p}{q} = \frac{k+1}{k} = 1 + \frac{1}{k}$$

$$\frac{5}{4} = \frac{1+4}{4} = 1 + \frac{1}{4}$$

$U(1)_{k} + N_f$



$$= \frac{2k - (k-1)}{k}$$

$q=1 \quad L(p,q), \qquad SL(2,\mathbb{Z})$

$$\begin{pmatrix} -q & * \\ p, & * \end{pmatrix},$$

$$= 2 - \cfrac{1}{\frac{k}{k-1}}$$

$$= 2 - \cfrac{1}{2 - \frac{k-1}{k-2}}$$

$$2(k-1) - (k-2)$$

$$2k - 2 - k + 2$$

$$\frac{23}{3} = \frac{24+2}{3} = 8 - \frac{1}{3}$$

$$\frac{5}{3} = \frac{6-1}{3} = 2 - \frac{1}{3}$$

$ST^{c_1} ST^{c_2} S \cdots T^{c_n} S.$

$$\frac{5}{4} = \frac{8-3}{4} = 2 - \frac{3}{4} = 2 - \cfrac{1}{\frac{4}{3}}$$

$$= 2 - \cfrac{1}{\frac{4-1}{3}} = 2 - \cfrac{1}{\frac{6-2}{3}}$$

$$= 6 - \cfrac{1}{2 - \cfrac{1}{\frac{3}{2}}}$$

$\boxed{L(p,1) \to ST^{p}S}$

$$\frac{1}{2} = \frac{2-1}{2} = 1 - 2$$

$$\frac{1}{2}$$

$$\frac{2n+1}{2} = \frac{2(n+1)-1}{2} \qquad \begin{bmatrix} n+1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= (n+1) - \frac{1}{2}$$

$$\frac{1}{2} = \frac{2-1}{2}$$

$$= 1 - \frac{1}{2}$$

$$\frac{4-1}{2} = 2 - \frac{1}{2}$$

$$\frac{5}{4} = 6 - \cfrac{1}{2 - \cfrac{1}{2 - \frac{1}{2}}}$$

$$\frac{1}{2} = \frac{2-1}{2} = 1 - \frac{1}{2}$$

$$\frac{1}{2} = 1 - \frac{1}{2}$$

$$= (n+1) - \cfrac{1}{1 - \frac{1}{2}}$$

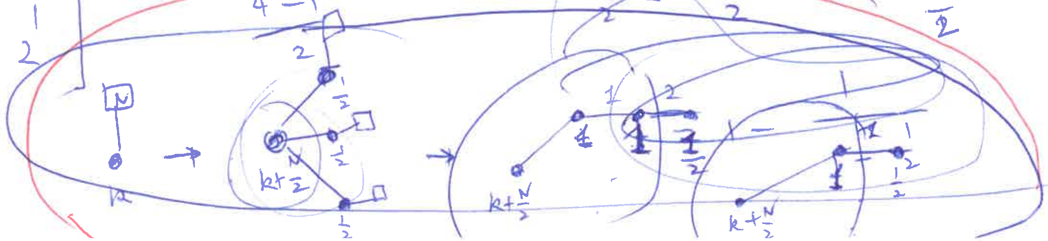
$$(n+1) - \frac{1}{1}$$

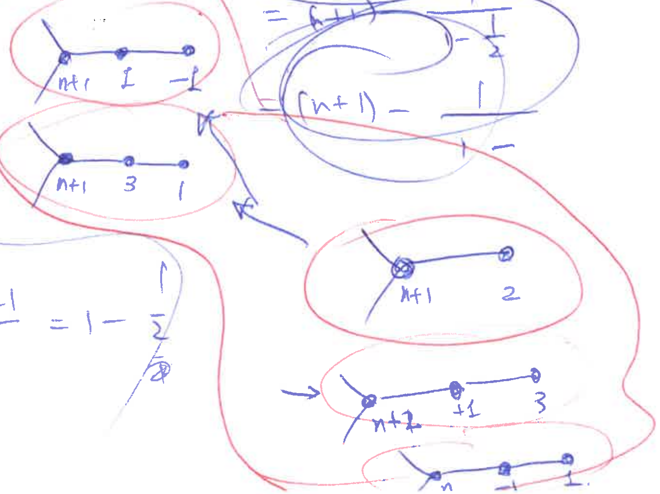
$$1 - \frac{1}{\frac{1}{2}}$$

$$\begin{bmatrix} 6 & 1 & & \\ 1 & 2 & 1 & \\ & 1 & 2 & 1 \\ & & 1 & 2 \end{bmatrix}$$

$$\frac{1}{2} = \frac{2-1}{2} = 1 - \cfrac{1}{\frac{2-1}{2}} = 1$$

$$\frac{1}{2} = \frac{2-1}{2} = 1 - \frac{1}{2}$$





2. Geometric setup

2.1. Branes and Riemann surfaces

We want to strengthen the conjecture made in [7] and clear up some technical details of this computation along the way. Let us therefore briefly review the geometric setup which we consider here.

Similarly to computations that were performed in [24] we want to compute the instanton series of the topological string A-model on non-compact Calabi-Yau spaces X, which are given as the total space of the fibration of the anti-canonical line bundle

$$\mathcal{O}(-K_B) \to B \tag{2.1}$$

over a Fano variety B. By the adjunction formula this defines a non-compact Calabi-Yau d-fold for $(d-1)$-dimensional Fano varieties. *Del Pezzo* surfaces are two-dimensional smooth Fano manifolds and they enjoy a finite classification. These consists of $\mathbb{P}^2$ and blow-ups of $\mathbb{P}^2$ in up to $n = 8$ points, called $\mathcal{B}_n$, as well as $\mathbb{P}^1 \times \mathbb{P}^1$.

As a result of mirror symmetry we are able to compute the amplitudes in the topological string B-model, where the considered geometry is given by

$$uv = H(e^p, e^x; z_I) \tag{2.2}$$

with $u, v \in \mathbb{C}$, $e^p, e^x \in \mathbb{C}^*$ and z_I are complex strucure moduli. Furthermore $H(e^p, e^x; z_I) = 0$ is the defining equation of a Riemann surface.

The analysis in the following relies heavily on the insertion of branes into the geometry and their behaviour when moved around cycles. Let us continue along the lines of [12] with the description of the influence branes have if we insert them into this geometry. In particular let us consider 2-branes. If we fix a point (p_0, x_0) on the (p, x)-plane these branes will fill the subspace of fixed p_0, x_0, where u and v are restriced by

$$uv = H(p_0, x_0). \tag{2.3}$$

The class of branes in which we are interested, corresponds to fixing (p_0, x_0) in a manner so that they lie on the Riemann surface, i. e.

$$H(p_0, x_0) = 0\,. \tag{2.4}$$

By fixing the position of the brane like this, the moduli space of the brane is given by the set of admissible points, meaning it can be identified with the Riemann surface itself.

Following from an analysis of the worldvolume theory of these branes, one can argue that the two coordiantes x and p have to be noncommutative. Namely, this means that they fulfill the commutator relation

$$[x, p] = g_s\,, \tag{2.5}$$

where g_s is the coupling constant of the topological string, which takes the role of the Planck constant.

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3d $\mathcal{N}=2$

$z \to q z$

$$\cancel{e^{\frac{2\pi}{2}\left(\frac{iQ}{2}-qz\right)^2}}\; S_b\left(\frac{iQ}{2}-qz\right) = \int d(py)\, e^{-\frac{i\pi}{2}p^2y^2}\, e^{2\pi i\left(\frac{iQ}{4}-qz\right)py}\, S_b\left(\frac{iQ}{2}-py\right)$$

$$= p\int dy\, e^{-\frac{i\pi p^2}{2}y^2 - 2\pi i\, qp\, zy + 2\pi i\frac{iQ}{4}py}\; S_b\left(\frac{iQ}{2}-py\right)$$

$$\times\, e^{(-)\frac{\pi i}{2}\left(\frac{iQ}{2}-qz\right)^2}$$

$S_b\left(\frac{iQ}{2}+qx+m_i\right)$

$z \to qz + m,$

$-\frac{\pi i}{2} q^2$

$$\begin{array}{c|c} & z \\ \hline z & [\;k\;] \end{array} \xrightarrow{ms} \begin{array}{c|cc} & z & y \\ \hline z & k+\frac{q^2}{2} & pq \\ y & pq & \frac{p^2}{2} \end{array}$$

$q=1,\ p=1$

$[k] \to \begin{bmatrix} k+\frac{1}{2} & 1 \\ 1 & \frac{1}{2} \end{bmatrix}$

$\begin{cases} q=1, & p=-1 \\ q=-1, & p=1 \end{cases}$

$[k] \to \begin{bmatrix} k+\frac{1}{2} & -1 \\ -1 & \frac{1}{2} \end{bmatrix}$

$q=-1,\ p=-1$

$[k] \to \begin{bmatrix} k+\frac{1}{2} & 1 \\ 1 & \frac{1}{2} \end{bmatrix}$

$S_b\left(\frac{iQ}{2}(1-r) - m'\right)$

$-\pi i\,(2qp$

q^2

<4

$\cancel{q \to 2,\ p=2}$

$p=\sqrt{2}$

$$\begin{array}{c|c} & z \\ \hline z & [\;k\;] \end{array} \xrightarrow{ms} \begin{array}{c|ccc} & z & y_1 & y_2 \\ \hline z & k+1 & & \\ y_1 & & & \\ y_2 & & & \end{array}$$

3d $\mathcal{N}=4$

$$\cancel{\int d\sigma\, e^{-2\pi i\sigma s}\frac{1}{\cosh(\sigma)}}$$

$$\boxed{\frac{1}{\cosh(z)} = \int dy\, e^{-2\pi i zy}\frac{1}{\cosh(y)}}$$

$$S_b\left(\frac{iQ}{2}-z\right)S_b\left(\frac{iQ}{2}+z\right) = e^{-\frac{i\pi}{2}\left[\left(\frac{iQ}{2}-z\right)^2+\left(\frac{iQ}{2}+z\right)^2\right]}\int dy_1\int dy_2\; e^{-\frac{i\pi}{2}y_1^2 - \frac{i\pi}{2}y_2^2}\, e^{2\pi i\left(\frac{iQ}{4}-z\right)y_1 + 2\pi i\left(\frac{iQ}{4}+z\right)y_2}\; S_b\left(\frac{iQ}{2}-y_1\right)S_b\left(\frac{iQ}{2}+y_2\right)$$

$$= e^{\frac{+i\pi}{2}\left[2\left(\frac{iQ}{2}\right)^2+2z^2\right]}\int dy_1\, dy_2$$

Looking at (2.11), we see that negative entries in the l-vectors lead to noncompact directions in M.

But we are going to do computations in the topological string B-model defined on the mirror W of M. We will now describe briefly how W will be constructed. Let us define $x_i := e^{y_i} \in C^*$, where $i = 1, \cdots, k+3$ are homogeneous coordinates. Using the charge vectors l^α, we define coordinates z_α by setting

$$z_\alpha = \prod_{i=1}^{k+3} x_i^{l_i^\alpha}, \quad \alpha = 1, \ldots, k. \tag{2.13}$$

These coordinates are called *Batyrev coordinates* and are chosen so that $z_\alpha = 0$ at the large complex structure point. In terms of the homogeneous coordinates a Riemann surface can be defined by writing

$$H = \sum_{i=1}^{k+3} x_i. \tag{2.14}$$

Using (2.13) to eliminate the x_i and setting one $x_i = 1$, we are able to parameterize the Riemann surface (2.14) via two variables, which we call $X = \exp(x)$ and $P = \exp(p)$. Finally, the mirror dual W is given by the equation

$$uv = H(e^x, e^p; z_I) \quad I = 1, \ldots, k. \tag{2.15}$$

3. The refinement

This was the story for the unrefined case, but we actually are interested in the refined topological string. Let us therefore introduce the relevant changes that occur when we consider the refinement of the topological string. According to [18], the partition function of the topological A-model on a Calabi-Yau X is equal to the partition function of M-theory on the space

$$X \times TN \times S^1 \tag{3.1}$$

where TN is a Taub-NUT space, with coordinates z_1, z_2. The TN is fibered over the S^1 so that, when going around the circle, the coordinates z_1 and z_2 are twisted by

$$z_1 \to e^{i\epsilon_1} z_1 \quad \text{and} \quad z_2 \to e^{i\epsilon_2} z_2. \tag{3.2}$$

This introduces two parameters ϵ_1 and ϵ_2 and unless $\epsilon_1 = -\epsilon_2$ supersymmetry is broken. But if the Calabi-Yau is non-compact we are able to relax this condition, because an addtional $U(1)_R$-symmetry, acting on X, exists.

General deformations in ϵ_1 and ϵ_2 break the symmetry between z_1 and z_2 of the Taub-NUT space in (3.1). As a result we find two types of branes in the refinement of the topological string. In the M-theory setup the difference is given by the cigar subspaces $\mathbb{C} \times S^1$ in $TN \times S^1$ of (3.1), which the M5-brane wraps.

The classical partition function of an ϵ_i-brane is now given by

$$\Psi_{i,\text{cl.}(x)} = \exp\left(\frac{1}{\epsilon_i} W(x)\right), \tag{3.3}$$

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$$\frac{1}{2}\left((\log x)^2\right) + 3\log x \log(-t)^3 + \log(-t)^3 \quad \log(-t)^3$$

$$\frac{1}{2}\left((\log y_1)^2\right) + 3\log y_1 \cdot \log y_2 + (\log y_3)^2$$

$$\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$$

$$(x;q)_\infty \sim e^{\frac{Li_2(x)}{\hbar} + \frac{1}{2}\log(1-x)}$$

$$\boxed{(3,1)} \qquad \boxed{(3,1)}$$

$$Z_{\hat{q}\text{-bar}} = \frac{z^n \prod_j (\alpha_j \sqrt{t/q}; t)_n}{(t;t)_n \prod_j (\beta_j \frac{t}{q}; t)_n} \longrightarrow Z_{\hat{q}\text{-bar}} = \frac{(z;t)_\infty \prod_j (z\beta_j \frac{t}{q}; t)_\infty}{\prod_j (z\alpha_j \sqrt{\frac{t}{q}}; t)_\infty}$$

$$= Pr(\alpha_i, \beta_i, q)$$

$$F_k(\alpha_i, \beta_i, q)$$

$$\frac{1}{y_1 y_2} = y_3$$

$$(\alpha; q)_n = \frac{(\alpha; q)_\infty}{(\alpha q^n; q)_\infty}$$

$q \to 0_+$

Pr F_k

NS   NS P_3

Pr $\rightarrow$ F_k moving D_3-brane ? ?

unknot :
$$Pr^{(a,q)} = a^{-\frac{r}{2}} q^{\frac{r}{2}} \frac{(a;q)_r}{(q;q)_r} \longrightarrow F_k(x,a,q) = \frac{(x, q; q)_\infty}{(xa; q)_\infty}$$

$$\boxed{x = q^r}$$

$$~~r = \log x~~ \qquad q = e^{\hbar} \qquad Pr(a,q) \qquad \boxed{x = q^r} \quad a^{-\frac{\log x}{\hbar}} \quad x^{\frac{1}{2}}$$

$$\log x = r \log q$$
$$= r\hbar$$

$$\boxed{r = \frac{\log x}{\hbar}}$$

$$\phi$$

$$F_k(x, a, q) = \frac{(-\sqrt{q})^{\text{kef } n^2} q^{\frac{n}{2}}}{(q;q)_n} \cdot \frac{\prod_{i=1} (\alpha_i; q)_n}{\prod_{j=1} (\beta_j; q)_n} = \frac{(-\sqrt{q})^{\text{kef } n^2} q^{\frac{n}{2}}}{~~(q;q)_\infty~~} \frac{(q \cdot q^n; q)_\infty}{~~(q;q)_\infty~~} : \frac{\prod_{i=1} (\alpha_i; q)_\infty}{\prod_{i=1} (\alpha_i q^n; q)_\infty}$$

$$\times \frac{\prod_{j=1} (\beta_j q^n; q)_\infty}{\prod_{j=1} (\beta_j; q)_\infty}$$

$$= (-\sqrt{q})^{\text{kef } n^2} q^{\frac{n}{2}} \frac{(qx; q)_\infty \prod_{i=1} (\beta_i x; q)_\infty}{\prod_{i=1} (\alpha_i x; q)_\infty} \qquad \boxed{x = q^n}$$

where $W(x)$ is the superpotential of the $\mathcal{N} = (2,2)$, $d = 2$ world-volume theory on the brane and which is identified with the p-variable in (2.15) as

$$W(x) = -\int^x p(y)\mathrm{d}y\,. \tag{3.4}$$

This is quite similar to (2.6) and still looks like the leading order contribution of a WKB expansion where only the coupling changed.

This suggests that the $\epsilon_{1/2}$-branes themselves also behave like quantum objects and if we have again say an ϵ_1-brane with only one point lying on the Riemann surface parameterized by (p, x) then the two coordinates are again noncommutative, i. e.

$$[x,p] = \epsilon_1 = \hbar\,. \tag{3.5}$$

We will show later that the free energy of the refined topological string can be extracted from a brane-wave function like this in a limit where we send either one of the ϵ-parameters to zero. The limit of ϵ_i to zero means that one of the branes of the system decouples. In the next section we will describe the relevant limit.

3.1. The Nekrasov-Shatashvili limit

In [9] the limit where one of the deformation parameters is set to zero was introduced. The free energy in this so called *Nekrasov-Shatashvili* limit is defined by

$$\mathcal{W}(\hbar) = \lim_{\epsilon_2\to 0} \epsilon_1\epsilon_2 F. \tag{3.6}$$

where $\mathcal{W}$ is the called the *twisted superpotential.* This $\mathcal{W}$ can be expanded in $\hbar$ like

$$\mathcal{W}(\hbar) = \sum_{n=0} \hbar^{2n}\mathcal{W}^{(n)} \tag{3.7}$$

where the $\mathcal{W}^{(i)}$ can be identified like

$$\mathcal{W}^{(i)} = F^{(i,0)} \tag{3.8}$$

with the free energy in the expansion (1.3).

Because we are only computing amplitudes in this limit, we present a convenient definition of the instanton numbers, tailored for usage in this limit. We define the parameters

$$\epsilon_L = \frac{\epsilon_1-\epsilon_2}{2}, \quad \epsilon_R = \frac{\epsilon_1-\epsilon_2}{2} \tag{3.9}$$

and accordingly

$$q_{1,2} = e^{\epsilon_{1,2}}, \quad q_{L,R} = e^{\epsilon_{L,R}}. \tag{3.10}$$

Using this definition the free energy at large radius has the following expansion

$$F^{hol}(\epsilon_1,\epsilon_2,t) = \sum_{\substack{j_L,j_R=0\\k=1}}^{\infty} \sum_{\beta\in H_2(M,\mathbb{Z})} (-1)^{2(j_L+j_R)} \frac{N^\beta_{j_Lj_R}}{k} \frac{\sum_{m_L=-j_L}^{j_L} q_L^{km_L} \sum_{m_R=-j_R}^{j_R} q_R^{km_R}}{2\sinh\left(\frac{k\epsilon_1}{2}\right)\,2\sinh\left(\frac{k\epsilon_2}{2}\right)} e^{-k\,\beta\cdot t} \tag{3.11}$$

Nov 28

$$Li_2(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^2}$$

$$Li_2(e^x) = \sum_{k=1}^{\infty} \frac{e^{xk}}{k^2}$$

$$e^{\frac{d Li_2(y)}{d \log y}} = \frac{1}{1-y}$$

$$e^{\frac{d Li_2(x)}{dx}} = \frac{1}{1-e^x}$$

$\log y = x$

$$\boxed{\left(-\sqrt{q}\, e^{-2\pi b\, x_i}\right) = y_i}$$

$1.8 \times 4 = 7.2$

$2.4 \times 3 = 7.2$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

8.6, 36

$$e^{\frac{\sigma_i}{\hbar}} \log y_i$$

$$y_i = |y_i|\, e^{2\pi i \theta_i}$$

$$\frac{dy}{y} \quad d(\log y)$$

2.4×3

$$e^{SW}$$

$2.4 \times 1.6 = 144$, 24, 384

$= 1 + SW + g^2 W^2 + \ldots$

$\log(-y_i)^{\frac{x_i}{-2\pi b}} = \log \sqrt{q}$

$d \log y_i$

$= x_i$

$$z = e^{\hbar}$$

2.20, 2.86, 5.06

$$e^{\frac{\tilde{w}^k}{\hbar}} \doteq \int_{\Gamma} \prod_i dx_i \ldots =$$

$$= e$$

$$\boxed{W^{eff} = \sum_i \left[Li_2(y_i) + \sigma_i^{eff} \log y_i \right] + \sum_{i,j} \left[\frac{k_{ij}^{eff}}{2} \log y_i \log y_j \right]}$$

$$e^x = \sum_{n=1}^{\infty} \frac{x^n}{n!}$$

$$e^{\frac{W_{class}}{\hbar}} = \int dy_i\, e^{\frac{\tilde{W}^{eff}}{\hbar}} = \int \prod \frac{dy_i}{y_i}\, e^{\frac{Li_2(y_i)}{\hbar}}\, y_i^{\frac{\sigma_i}{\hbar}} \cdot e^{\frac{k_{ij}^{eff}}{2\hbar} Y_i Y_j}$$

$$= \int \prod_i dY_i\, e^{\frac{\sum Li_2(e^{Y_i})}{\hbar}}\, e^{Y_i \frac{\sigma_i^{eff}}{\hbar}} \cdot e^{\ldots}$$

$$\frac{\sigma_i^{eff}}{\hbar} Y_i$$

$$e^{ab} = (e^a)^b$$

$$\boxed{y_i = e^{Y_i}}$$

$$\frac{e^{Y_i}\, dY_i}{e^Y}$$

$k_{ij}^{eff}\, \log y_i \log y_j$

$$\boxed{x_i = e^{Y_i}}$$

$Y_i = \log X_i$

$d \log x_i = \frac{dx}{x_i}$

$x_i = e^{Y_i}$

$$\int \prod_i \frac{dX_i}{X_i}\, e^{\sum Li_2(X_i)}$$

$$X_i^{\frac{\sigma_i^{eff}}{\hbar}} \quad X_i^{\frac{k_{ij}}{2\hbar} \log x_j}$$

$$x_i^{\frac{k_{ij}}{2\hbar} \log x_j + \frac{\sigma_i^{eff}}{\hbar}}$$

$$(e^{Y_i})^{\frac{k_{ij}^{eff}}{2\hbar} \log X_j}$$

$$= \int \prod_i dY_i\; e^{\frac{\sum Li_2(e^{Y_i}) + \sigma_i^{eff} Y_i + \frac{k_{ij}^{eff}}{2} Y_i Y_j}{\hbar}}$$

Nov 28

$$W_{disc} \overset{?}{=} \int dx_i \, \tilde{W}_{eff}$$

$$y_i = e^{x_i}$$

$$\cancel{W}_{dis} = \int \prod_i dx_i \Big[\sum_i Li_2(e^{x_i}) + \cancel{\theta_i} \, y_i^{eff} x_i + \frac{k_{ij}^{eff}}{2} x_i x_j \Big]$$

$$=$$

Jan 3

? How to extend CY_3 (5d) to CY_4 (3d) ?

$A^{(3)} \wedge F^{(4)} \wedge \tilde{F}^{(4)}$ $F^{(4)} = v^{\gamma} \omega_{\gamma}^{2,2}$

M2. $F^{(4)} = dA^{(3)} = dA^{\alpha} \wedge \omega_{\alpha}^{1,1}$

$A^{(3)} = A^{\alpha} \wedge \omega_{\alpha}^{1,1}$

$A^{(3)} \wedge F^{(4)} \wedge \tilde{F}^{(4)} = v^{\gamma} A^{\alpha} \wedge \omega_{\alpha}^{1,1} \wedge dA^{\beta} \wedge \omega_{\beta}^{1,1} \wedge \omega_{\gamma}^{2,2}$

$= A^{\alpha} \wedge F^{\beta} \quad v^{\gamma} \int \omega_{\alpha}^{1,1} \wedge \omega_{\beta}^{1,1} \wedge \omega_{\gamma}^{2,2}$

$= A^{\alpha} \wedge F^{\beta} \cdot v^{\gamma} K_{\alpha\beta\gamma}$

3pt function in CY_4

$C^{(3)} \wedge dB^{(2)}$

M5 brane on 3-fold M_3

3d $N=2$

M5-brane action

$6-3 = 3d$ (M_3)

$M_3 \sim S^2 \times S^1$

$S_{M5} = \int (C^{(6)} + C^{(3)} \wedge F^{(3)})$

$F^{(3)} = H^{(3)} + C^{(3)} = dB^{(2)} + C^{(3)}$

$A^{(3)} \wedge (dB + A^{(3)})$

$$\boxed{S_{M5} = \int (C^{(6)} + C^{(3)} \wedge F^{(3)})}$$

M5 → $C^{(3)} = A^{(1)} \wedge \omega^{1,1}$

$C^{(3)} \wedge C^{(3)} = A_{\alpha}^{(1)} \wedge \omega_{\alpha}^{1,1} \wedge A^{(1)} \wedge \omega^{(1,1)}$

M3 $A_{\alpha}^{(1)} \int_{S^2} \omega_{\alpha}^{(1,1)} \wedge$

$C^{(3)} \wedge dB^{(2)}$

$\int_{S^2} B^{(2)} = b$

$C^{(3)} = A^{(1)} \wedge \omega^{1,1}$

$A^{(1)} \wedge \omega^{1,1} \wedge dB^{(2)} \wedge \omega^{1,1}$

$\to dA^{(1)} \wedge B^{(2)}$

$\to dA^{(1)} \wedge \int_{S^2 \times S^2} B^{(2)} \wedge \omega^{(1,1)}$

$\to dA^{(1)} \wedge A^{(1)} \cdot \int_{S^2} \omega^{(1,1)}$

in terms of BPS numbers $N^{\beta}_{j_L j_R}$.

By a change of basis of the spin representations

$$\sum_{g_L,g_R} n^{\beta}_{g_L,g_R} I_L^{g_L} \otimes I_R^{g_R} = \sum_{j_L,j_R} N^{\beta}_{j_L,j_R} \left[\frac{j_L}{2}\right]_L \otimes \left[\frac{j_R}{2}\right]_R \tag{3.12}$$

we introduce the instanton numbers $n^{\beta}_{g_R,g_L}$, which are more convenient to extract from our computations. With the sum over the spin states given by the expression

$$\sum_{m=-j}^{j} q^{km} = \frac{q^{j+\frac{k}{2}} - q^{-j-\frac{k}{2}}}{q^{\frac{k}{2}} - q^{-\frac{k}{2}}} = \chi(q^{\frac{k}{2}}) \tag{3.13}$$

we write down the relation between $N^{\beta}_{j_L j_R}$ and the numbers $n^{\beta}_{g_R,g_L}$ defined in (3.12) explicitly [21, 22]

$$\sum_{j_L,j_R} (-1)^{2(j_L+j_R)} N^{\beta}_{j_L j_R} \chi(q_L^{\frac{k}{2}})\chi(q_R^{\frac{k}{2}}) = \sum_{g_L,g_R} n^{\beta}_{g_L,g_R} (q_L^{\frac{1}{2}} - q_L^{-\frac{1}{2}})^{2g_L} (q_R^{\frac{1}{2}} - q_R^{-\frac{1}{2}})^{2g_R} . \tag{3.14}$$

Since we do not consider the full refined topological string we want to see how this expansion looks like in the Nekrasov-Shatashvili limit. Writing (3.11) in terms of $n^{\beta}_{g_L,g_R}$ and taking the Nekrasov-Shatashvili limit (3.6), we find

$$\mathcal{W}(\hbar, t) = \hbar \sum_{\substack{g=0 \\ k=1}}^{\infty} \sum_{\beta \in H_2(M,\mathbb{Z})} \frac{\hat{n}^{\beta}_g}{k^2} \frac{(q^{\frac{k}{4}} - q^{-\frac{k}{4}})^{2g}}{2\sinh\left(\frac{k\hbar}{2}\right)} e^{-k\beta \cdot t} \tag{3.15}$$

where $\hbar = \epsilon_1$ and

$$\hat{n}^{\beta}_g = \sum_{g_L+g_R=g} n^{\beta}_{g_L,g_R} . \tag{3.16}$$

3.2. Schrödinger equation from the β-ensemble

In [12] the authors described the behavior of branes by analyzing the relevant insertions into the matrix model description of the topological string B-model. In [10] a conjecture has been made about a matrix model description of the refined topological B-model, which we now want to use as described in [7] to derive a Schrödinger equation for the brane-wavefunction of an ϵ_1 or ϵ_2-brane. This matrix model takes the form of a deformation of the usual matrix model, describing the unrefined topological string where the usual Vandermonde-determinant is not taken to the second power anymore, but to the power 2β where

$$\boxed{\beta = -\frac{\epsilon_1}{\epsilon_2}.} \tag{3.17}$$

This clearly has the unrefined case as its limit, when $\epsilon_1 \to -\epsilon_2$. Matrix models of this type are called β-ensembles.

The partition function of this matrix model is

$$\boxed{Z = \int \mathrm{d}^N z \prod_{i<j} (z_i - z_j)^{-2\epsilon_1/\epsilon_2} e^{-\frac{2}{\epsilon_2}\sum_i W(z_i)}.} \tag{3.18}$$

Jan 2.

Chern-Simons   AD theory (A_n, D_n)

(A_1, A_n)

$\leftrightarrow$ ~~sang~~ sang

~~Chern~~

$k_{ab}\,\Sigma^a \vee^b$

$\vee^a = b.$ $\leftrightarrow$

3 : 10   4 : 00

4d $\mathcal{N}=2$ $\xrightarrow{/S^1}$ 3d $\mathcal{N}=4$ T_{3d}

mirror

3d $\mathcal{N}=4$ $\tilde{T}_{3d}$.

$\oint A^a \wedge dA^b = d(A^a \wedge A^b)$

$0 = d(A^a \wedge A^b) = dA^a \wedge A^b + (-)^1 A^a \wedge dA^b$

$\boxed{dA^a \wedge A^b} = A^a \wedge d\boxed{A^b}$

$0 \Rightarrow$

$F^a \wedge A^b = A^a \wedge F^b$

symmetric

$\int k_{ab}\, A^a \wedge F^b \Rightarrow k_{ab} = k_{ba}.$

M5

6

Ω $2 + \mathbb{R}^4$

$\Sigma \times \mathbb{R}^1 \times \mathbb{R}^3$

4d

$\to k_{ij}$

$\omega^i \wedge \omega^j = c_i\, c_j$

$i j = \int_{B} \leftarrow 2$ mfd

Ω i, B

in 5d   CS $A^i \wedge F^j \wedge F^k$

$I_{CS} = c_{ijk}$

$$R^\lambda{}_{\nu\rho\sigma} = \partial_\rho \Gamma^\lambda_{\nu\sigma} - \partial_\sigma \Gamma^\lambda_{\nu\rho} + \Gamma^\lambda_{\rho\eta}\Gamma^\eta_{\nu\sigma} - \Gamma^\lambda_{\sigma\eta}\Gamma^\eta_{\nu\rho}$$

$$R_{\nu\sigma} = R^\rho{}_{\nu\rho\sigma} = \partial_\rho \Gamma^\rho_{\nu\sigma} - \partial_\sigma \Gamma^\rho_{\rho\nu} + \Gamma^\rho_{\rho\eta}\Gamma^\eta_{\nu\sigma} - \Gamma^\rho_{\sigma\eta}\Gamma^\eta_{\nu\rho}$$

$$= \cancel{\partial_\rho \partial_\nu \log\sqrt{-g}} - \partial_\sigma\partial_\nu \log\sqrt{-g} + (\partial_\eta \log\sqrt{-g})\,\Gamma^\eta_{\nu\sigma} - \Gamma^\rho_{\sigma\eta}\Gamma^\eta_{\nu\rho}$$

$\partial_\mu D^\mu$. $\quad \Gamma^\mu_{\mu\nu} = \partial_\nu \log\sqrt{-g}$

$$= \partial_\rho \Gamma^\rho_{\nu\sigma} - \partial_\sigma \partial_\nu \log\sqrt{-g} + (\partial_\eta \log\sqrt{-g})\Gamma^\eta_{\nu\sigma} - \Gamma^\rho_{\sigma\eta}\Gamma^\eta_{\nu\rho}$$

$\cancel{\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\lambda}}$

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\lambda}(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu})$$

$$g^{\mu\nu} = \begin{bmatrix} 1 & & & \\ & \frac{1-kr^2}{a^2(t)} & & \\ & & \frac{1}{a^2(t)\,r^2} & \\ & & & \frac{1}{a^2(t)\,r^2\sin^2\theta} \end{bmatrix}$$

(t, r, θ, φ)

$$\sqrt{-g} = \sqrt{\frac{a^2(t)}{1-kr^2}\, a^2(t)\, r^2\, a^2(t)\, r^2 \sin^2\theta}$$

$g^{\mu\nu}$

$$\boxed{\sqrt{-g} = \frac{r^2 \sin\theta\, a^3(t)}{\sqrt{1-kr^2}}}$$

$$\boxed{\log\sqrt{-g} = 2\log r + \log\sin\theta + 3\log a(t) - \frac{1}{2}\log(1-kr^2)}$$

$\cancel{\partial_\sigma g_{\mu\nu} = \partial_\sigma g_{\mu\nu}\ \delta^\mu{}_\nu}$

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\rho}(\partial_\mu g_{\rho\nu} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu})$$

$\partial_\mu g_{\lambda\nu}$

$g^{00} = 1$

$\cancel{\frac{1}{2} g^{\rho\lambda}\,\delta^\rho{}_\lambda\,(\partial_\mu g_{\lambda\nu}}$

$\log\sqrt{-g} =$

$$\Gamma^0_{\mu\nu} = \frac{1}{2}[\partial_\mu g_{0\nu} + \partial_\nu g_{0\mu} - \partial_0 g_{\mu\nu}]$$

$\Gamma^\mu_{\mu 1} =$

$\cancel{\Gamma^t_{tt} \quad \Gamma^t_{tr} \quad \Gamma^t_{t}}$

$$\Gamma^0_{0\nu} = \frac{1}{2}[\partial_0 g_{0\nu} + \partial_\nu g_{00} - \partial_0 g_{0\nu}] = \frac{1}{2}[$$

$\cancel{\Gamma^\mu_{\mu\theta}} =$

$\cancel{\Gamma^0_{0\nu} = \frac{1}{2}[\partial_0 g_{0\nu}}$

$$\boxed{\Gamma^\mu_{\mu 0} = \partial_0 \log\sqrt{-g} = \frac{3\dot a}{a}}$$

$$\Gamma^\mu_{\mu\nu} = \partial_\nu \log\sqrt{-g}.$$

$$\Gamma^\mu_{\mu\nu} = \frac{1}{2} g^{\mu\mu}(\cancel{2}\partial_\mu g_{\mu\nu} + \cancel{\partial_\nu g_{\mu\mu}} - \cancel{\partial_\mu g_{\mu\nu}})$$

$$= \frac{1}{2} g^{\mu\mu}\partial_\mu g_{\mu\nu}. \qquad \Gamma^0_{00} = \frac{1}{2}\cdot\partial_0 g_{00} = 0$$

$$\Gamma^r_{r\theta} = \frac{1}{2} g^{rr}\partial_r g_{r\theta} = \frac{1}{2}\,\frac{1-kr^2}{a^2}.$$

Jan 07

? from 5d $N=1$ to 3d $N=2$

CY_4

In 5-d

$$C_{ijk} = \frac{k_a}{6} d^a_{ijk} + \frac{|e\cdot\phi|^3}{12} - \frac{|w\cdot\phi|^3}{12}$$

$$\int A^i \wedge F^j \wedge F^k$$

$$\int \Omega_{ij} A^i \wedge F^j$$

$$\Sigma_i \wedge \Sigma_j = -\Omega_{ij}$$

$$\frac{-1}{q} = a_1 - \frac{1}{a_2 - \frac{1}{\cdots - \frac{1}{a_n}}}$$

from 2

5d SCFT

$d=11$ CY_3

$11 - 6 = 5$ d

$d = 11$ CY_4

$11 - 8 = 3$ d $N=2$

3d $N=2$

from 2

CY_4

4 supercharges

$F=H_3=dB$

6d

$\int B_2 \wedge F_2 \wedge F_2$

$A_i = \int_{S^2} B_{\mu\nu} dx^\nu$

M-theory 11d SUGRA

$F_4 = dA_3$

$$\frac{1}{12\kappa_{11}^2}\int A_3 \wedge F_4 \wedge F_4$$

$\int A_4 \wedge F_4 \wedge F_4$

$11d - 6 = 5d$ (CY_3)

$$S^{5d}_{CS} = C_{ijk}\int A_i \wedge F_j \wedge F_k$$

$$S^{3d}_{CS} = \int_{\beta_L} S^{5d}_{CS}$$

$$= C_{ijk}\int A_i \wedge F_j \wedge \int_{\beta_L} F_k$$

$$+ C_{ijk}\int A_i \wedge F_k \wedge \int_{\beta_L} F_j$$

$$= \sum_k C_{ijk}\, \Omega_{Lk} \int A_i \wedge F_j$$

$$+ \sum C_{ijk}\, \Omega_{Lj} \int A_i \wedge F_k$$

?

$$= 2\sum_k C_{ijk}\, \Omega_{Lk} \int A_i \wedge F_j$$

$11d - 4 = 6d$

CY_2

$11d - 8 = 3d$

C_{ijkl}

Ω_{lk}

6d $\xrightarrow{\int_{S^2}}$ 5d

$A^i = \int_{S^2} B^i_{\mu\nu}$

Warszawa, 17 grudnia 2021

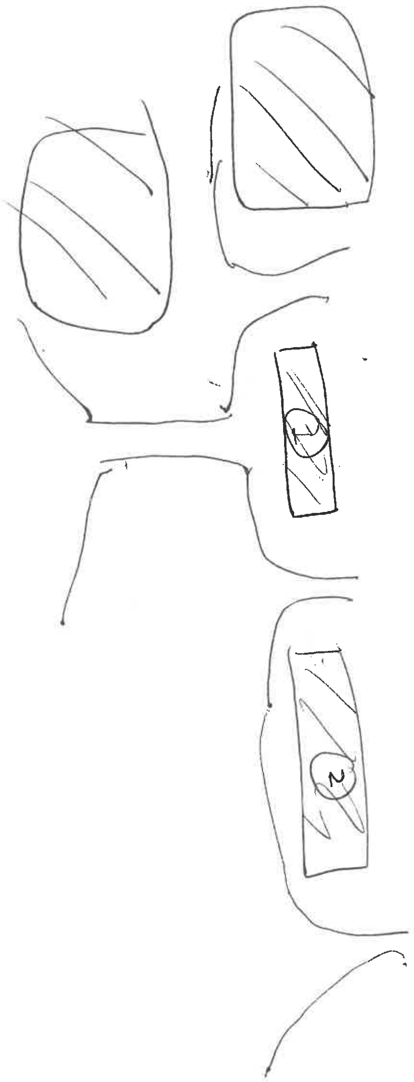
Dziekan Wydziału Fizyki UW
prof. dr hab. Dariusz Wasik

PODANIE

Szanowny Panie Dziekanie,

Proszę o zmianę katedry, której jestem członkiem jako ~~pracownik~~ doctorant Instytutu Fizyki Teoretycznej, i przypisanie mnie do „Katedry kwantowej fizyki matematycznej".

Z poważaniem,



Jan 2

$$k_{ij} = \frac{1}{2\pi}\int F_4 \int_{B_4} w_i \wedge w_j$$

3 d

$F_j = dA_j$

$$S_{cs} = k_{ij} \int A_i \wedge F_j$$

background flux

$$k_{ij} = \frac{1}{2\pi}\int_{CX_4} F_4 \wedge w_i \wedge w_j$$

$w_i \in H^{1,1}(X, \mathbb{Z})$

$A_3 = A_1^i \wedge w^i$

$F_4 = F_2^i \wedge w_j$

11 d − 8 d = 3 d

CY_4,

$i = 1, \ldots h^{1,1}$,

$$\int_{11d} A_3 \wedge F_4 \wedge F_4 = \int_{3d} A_3 \wedge$$

M2-brane $A_3 = A_1 \wedge w_a$

$F_4 = dA_3 = d(A_1 \wedge w_a)$

$= \boxed{dA_1^i \wedge w_i} - A_1^i \wedge dw^i$

$= \boxed{F^j \wedge w^j}$

$A_3 \wedge F_4 \wedge G_4 = A_1^i \wedge w^i \wedge F^j \wedge w^j \wedge G_4$

$= A^i \wedge F^j \wedge G \wedge w^i \wedge w^j$

11d. in M-theory

$F_4 = dA_3$

$$S_{cs} = \int_{11d} A_3 \wedge F_4 \wedge F_4$$

12d in F-theory.

$$S_{cs} = \int_{12d} F_4 \wedge F_4 \wedge F_4.$$

?

6d

$$S_{cs} = \Omega_{ij}\int_{6d} B_2^i \wedge F_2^j \wedge F_2^j. \quad ?$$

$A_\mu = \int_{S^1} B_{\mu\nu}\, dx^\nu.$

$$S_{cs} = \int_{11d} A^i \wedge F^j \wedge F^k \int_{CY_3} w_i \wedge w_j \wedge w_k.$$

$$= C_{ijk} \int_{5d} A^i \wedge F^j \wedge F^k$$

5d

$$S_{cs} = C_{ijk}\int_{5d} A_i \wedge F_j \wedge F_k.$$

$A_3 = A_1^i \wedge W_i$, $F_4 = dA_3 = dA_i^i \wedge w_i - A_i \wedge dw_i$

$dw_i = ?$

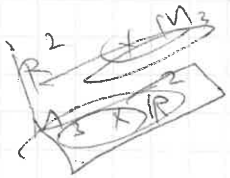
$A_3 \wedge F_4 \wedge F_4 = (A_3 \wedge dA_3) \wedge dA_3$

$= A_1^i \wedge W_i \wedge (dA_j \wedge w_j - A_1^j \wedge dw_j) \wedge (dA_k \wedge w_k - A^k \wedge dw_k)$

$= A_1^i \wedge W_i \wedge (F_j \wedge w_j - \cancel{A^j \wedge dw_j}) \wedge (F_k \wedge w_k - \cancel{A^k \wedge dw_k})$

$= A_1^i \wedge F^j \wedge F^k \boxed{\wedge w_i \wedge w_j \wedge w_k} + \ldots$

$A = A + i\phi$



$\cancel{SO(3)_E \times SO(3)_R}$

$SO(3)'_E \subset SO(3)_E \times SO(3)_R$

$L(p,1)$

$= S^3_{-p}(\circlearrowleft)$

$U(1)_E \times SO(3)_E \supset SO(3)'_E$

$U(1)_R \times SO(3)_R,$

$\mathbb{R}^2 \times M_3$

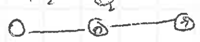
$\mathbb{R}P^2$ on

$U(1)_E \times U(1)_R \times SO(3)'_E$

R-sym.

$-p$

$-p+\frac{1}{2}$ $\frac{1}{2}$



$SO(3) \quad \boxed{SO(3) \quad SO(2)} \times SO(3)_R,$

$\mathbb{R}^3 \times M_3 \quad SO(5)$

D2 on X_3 $\xrightarrow[\text{strong}]{\text{LR}}$ M2 probing on X_4

coupling $g^2 \to \infty$ gauge kinetic term $\to 0$

N M3 probing M4, $\rightsquigarrow$ quiver CS theory.

D2 on C

$k = \int_C F_{RR}, = \sum_j q_j \# (D_j \cap C) = \sum_j q_j \alpha_j^C$

$[F_{RR}] = \sum_j q_j [D_j]$

$\# (D_j \cap C) = \alpha_j^C$

vertex change $\int F_\alpha = \# (D_\alpha \cap C)$

$U(1)_\alpha$

$k = f + \frac{1}{2}$

$f(x) = e^{\tilde{W}(x)}$

$\frac{df}{dx} = e^{\tilde{W}(x)} \frac{d\tilde{W}(x)}{dx}$

$f(x) = e^{\tilde{W}(x_0)} + e^{\tilde{W}(x_0)} \frac{d\tilde{W}(x_0)}{dx} (x - x_0) + \frac{i\pi}{2}$

$+ e$

$f'(x) = \left(e^{\tilde{W}(x)} \frac{d\tilde{W}}{dx}\right)$

$= e^{\tilde{W}(x)} \left(\frac{d\tilde{W}}{dx}\right)^2 + e^{\tilde{W}(x)} \frac{d^2 W}{dx^2}$

mesom

Seiberg dual

singlet

Seiberg dual

monopole op.

singlet

$b_{ij}^{(0)} = \frac{1}{2} + k$

$y^{\mathrm{eff}}(0) = 2b\pi\tilde{\zeta} + i\pi(1 - b(Q))k$

$y_j = (-)^{\frac{1}{2}+k} e^{2b\pi\tilde{\zeta}} e^{i\pi(1-b(Q))k}$

$= i(-)^k e^{2b\pi\tilde{\zeta}}$

$e^{(-)^{\frac{1}{2}}} e^{i\pi k} e^{-i\pi b Q k}$

$(-)^k \quad q^{-\frac{k}{2}}$

1.6

1.76

2.8 $(\pi \approx \pi)$

2.4, 2.5

2π

$q = e^{2\pi i b Q}$

$= i(-)^k e^{2\pi b\tilde{\zeta}} (-)^{\frac{1}{2}} (-)^k q^{-\frac{k}{2}}$.

$e^{\tilde{W}(x_0)} + e^{\tilde{W}(x_0)} \frac{d^2 W}{dx^2} (x - x_0)$

$\frac{1}{2} + k$

$= \frac{1}{2} + f + \frac{1}{2} = f + 1$

$\pm \left(\frac{1}{2} + k\right) = \frac{f}{2} + \frac{1}{2}$

$\frac{f}{2} + \frac{1}{2} = k + \frac{1}{2}$

$$\cancel{f(x) = e^{\tilde{W}_{ff}(x)}}$$

$$\cancel{\frac{d^2 f(x)}{d \log y_i \, d \log y_j} = \left(k_{ij} + \left(\sum_{\alpha} k_{i\alpha} \cdot \log y_\alpha \right) \cdot \left(\sum_{\beta} k_{j\beta} \log y_\beta \right) \right) f(x)}$$

$\delta(\alpha x) =$

$\delta(k) = \frac{1}{2\pi} \int dx\, e^{ikx}$

$\delta(k) = \int dx\, e^{i2\pi kx}$

$\delta(\alpha x) = \frac{\delta(x)}{|\alpha|}$

$\int_{-\infty}^{\infty} e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}$

Sulkowski

$\begin{pmatrix} p & 2 \\ 3 & 4 \end{pmatrix}$

$4 - 6$

$- e$

$2b\pi \left(\frac{i\alpha}{2} - y_2\right)$

$y \to \frac{i\alpha}{2} - \frac{-\log[-Y]}{2b\pi}$

Jordan decomposition

$A = P \Lambda P^{-1}$

$\Lambda = \begin{pmatrix} \lambda_1 & \dots & \lambda_n \end{pmatrix}$

Orthogonal matrix

$A^T = A^{-1}$

Cholesky decomposition

$A = L L^T$

$x \cdot A \cdot x^T$

(x_1, x_2)

$P\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$\begin{pmatrix} ax_1 + bx_2 \\ cx_1 + dx_2 \end{pmatrix}$

$x P \Lambda P^{-1} P^T x^T$

$x P^{-1} P \Lambda P^{-1} P x$

$(x_1, x_2) \cdot \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

$= (ax_1 + bx_2 \quad cx_1 + dx_2)$

$P P^T$

$① \rightleftarrows \boxed{2}$.

cs,

$k = 0$,

$$Z = \int dz \, e^{2\pi i \eta z} \, F_{-\phi_1}(z+m_1) \, F_{-\phi_2}(z+m_2)$$

$$F_{-\phi_1}(z+m_1) = \int dy_1 \, \cancel{e^{2\pi i y_1 p}} \, e^{-2\pi i y_1 (z+m_1)} \, F_{\phi_1}(y_1)$$

$$F_{-\phi_2}(z+m_2) = \int dy_2 \, e^{-2\pi i y_2 (z+m_2)} \, F_{\phi_2}(y_2)$$

$$Z = \int dz \int dy_1 \int dy_2 \, e^{2\pi i \eta z} \, e^{-2\pi i y_1 (z+m_1)} \, e^{-2\pi i y_2 (z+m_2)} \, e^{-\pi i k z^2},$$

$$\times F_{\phi_1}(y_1) \, F_{\phi_2}(y_2)$$

$$= \int dy_1 \, dy_2 \, F_{\phi_1}(y_1) \, F_{\phi_2}(y_2) \, \cancel{\int dz} \cdot e^{-2\pi i y_1 m_1} \, e^{-2\pi i y_2 m_2} \, \cancel{x}$$

$$y_1 + y_2 + \eta = 0$$

$$\times \boxed{\int dz \, e^{2\pi i \eta z} \, e^{-2\pi i y_1 z} \, e^{-2\pi i y_2 z}} \doteq \frac{i \, e^{-2\pi i z (y_1 + y_2 + \eta)}}{2\pi (y_1 + y_2 + \eta)} \, \delta(y_1 + y_2 + \eta), \quad \boxed{y_2 = \eta - y_1}$$

$$= \cancel{\int dy_1 \, dy_2 \, F_{\phi_1}(y_1) \, F_{\phi_2}(y_2) \, e^{-\pi i z (}} \quad \cancel{e^{-2\pi i m_2 (\eta - y_1)}}$$

$$= \int dy_1 \, F_{\phi_1}(y_1) \, F_{\phi_2}(\eta - y_1) \, \underbrace{e^{-2\pi i y_1 m_1} \, e^{-2\pi i m_2 (\eta - y_1)}}_{e^{-2\pi i y_1 (m_1 - m_2)} \, e^{-2\pi i m_2 \eta},}$$

$$= \cancel{e^{-2\pi i m_2 \eta}} \, \cancel{\int d y_1 F} \, \int dy_1 \, F_{\phi_1}(y_1) \, F_{\phi_2}(\eta - y_1) \, e^{-2\pi i y_1 (m_1 - m_2)}$$

For $\exists\, N = 4$, $k = 0$, and $k \neq 0$ ~~another~~ have

very different mirror duals theories

$k = 0$

$$① \rightleftarrows \boxed{N} \longleftrightarrow \underbrace{① \rightleftarrows ① \rightleftarrows \cdots \rightleftarrows ①}_{N-1}$$

$k \neq 0$

$$① \rightleftarrows \boxed{N} \longleftrightarrow ① \rightleftarrows \boxed{1}, \; \ldots, \; ① \rightleftarrows \boxed{1}$$

real, sym.

For a symmetric matric S

$$\boxed{S = Q \Lambda Q^T}$$

S: symmetric

Q: orthogonal

Λ: diagonal (real)

Eigendecomposition

(spectral decomposition)

real

real positive - definite symmetric matrix A, $\boxed{A = L L^T}$

Cholesky decomposition

$S = P \Lambda P^T$

Organize$[P^T] = Q^T$

$(O[P^T])^T = Q$

$Q^T S Q = \Lambda$

$A^T A = I$,

A 正交矩阵

$y = Px$ 正交变换

P 为 正交矩阵

$\frac{\Lambda}{4} \begin{pmatrix} -\sqrt{2} & 2-\sqrt{2} \\ +\sqrt{2} & \sqrt{2}+2 \end{pmatrix}$

$(-)\sqrt{2} \begin{pmatrix} +1 & -\sqrt{2}+1 \\ -1 & -\sqrt{2}+1 \end{pmatrix}$

$\begin{pmatrix} -\sqrt{2}-1 & 1 \\ \sqrt{2}-1 & 1 \end{pmatrix}$

$\begin{pmatrix} \frac{(-)\sqrt{2}+1}{\sqrt{2}-1} & 1 \\ \frac{\sqrt{2}+1}{\sqrt{2}} & \frac{\sqrt{2}+1}{\sqrt{2}+1} \end{pmatrix}$

$\frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$2-1$

$(-)\frac{(\sqrt{2}+1)^2}{\cancel{\sqrt{2}-1}}$

$(2+1-2\sqrt{2})$

$3-2\sqrt{2}$

$1 + f = k$

$\frac{1}{2} + k = f + 1$

If A is symmetric, then $B^T A B$ is also symmetric

if A is symmetric, A^{-1} is also symmetric

$\cancel{BB^T = BB^T = I}$

$A = \begin{pmatrix} \pm\sqrt{\lambda_1} & 0 \\ 0 & \pm\sqrt{\lambda_2} \end{pmatrix}$

$\cancel{A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}}$, $A^T A = A^2 = \Lambda I$

$\begin{pmatrix} a & b \\ b & c \end{pmatrix}\begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} a^2+b^2 & ab+bc \\ ab+bc & b^2+c^2 \end{pmatrix}$

$(-)^f q^{-\frac{k}{2}} e^{2bz\tilde{\xi}}$

$\int dy_2$

$\int e^{ix}\, dx = \int e$

$= \int e^{i(-i\xi)x}\, dx.$

$= \delta(-i\xi) \rightarrow \xi = 0$

$\cancel{\delta(Q + i q_1)}$

$\delta(Q + q_1 i s_1 + q_1 i s_1 + i x_2 k_{12}) = 0$

$\begin{pmatrix} a & b & c \\ b & d & f \\ c & f & e \end{pmatrix}$

$(-)^{k-\frac{1}{2}} q^{-\frac{k}{2}} e^{2bz\tilde{\xi}}$

$= \cancel{(-)^f} q^{-\frac{\cancel{f}+\frac{1}{2}}{2}} e^{2bz\tilde{\xi}}$

$= \cancel{(-)^{f+1}} (-)^1 q^{-\frac{\cancel{f}+1}{2}} x$

$f = k - \frac{1}{2}$

$f + \frac{1}{2}$

$2 k_{12}^2 \quad (1 + 2k_{11}) k_{22}$

$2 k_{12}^2 = (1 + 2k_{11}) k_{22}$

Jordan Decomposition $[A]$

$= \{P, \Lambda\}$

$P \Lambda P^{-1} = A$

$\cancel{q^{-f}} e^{2bz\tilde{\xi}}$

$(-) q^{\frac{1}{4}} e^{2bz\tilde{\xi}} = (-) q^{-\frac{1}{4}} x$

$= x$

$-\frac{1}{4}$

$q^{\frac{1}{4}} q^{\frac{1}{2}}$

if A is integral and symmetric

$\cancel{A = \begin{pmatrix} a & b & c \\ b & d & f \\ c & f & e \end{pmatrix} \text{ if } A^T A = A^2 = \Lambda I}$

$A = \begin{pmatrix} \pm\sqrt{\lambda_1} & 0 & 0 \\ 0 & \pm\sqrt{\lambda_2} & 0 \\ 0 & 0 & \pm\sqrt{\lambda_3} \end{pmatrix}$

I

$\sum k_{12}^2 - (1 + 2k_{11}) k_{22}$

$k_{11} = -\frac{1}{2}$

$2k_{12}^2 + k_{21} = 0$

$k_{12} = 0$

$1 \quad -1$

$y_2 \neq \xi_2$

orthogonal matrix $\quad Q^T Q = QQ^T = I$

$Q^T = Q^{-1}$

$4 + 2 + 2 + 4 + 4$

4

$\left(\frac{16\rho + 3\alpha}{2}\right)^n$

$\frac{3 - 2k}{2 + 4k}$ ~~$+ 1 + 2k$~~

$\frac{2k-1}{1+2k}$

$(1, 1, 1, 1, 2, 2)$,

$\frac{3\alpha}{2} +$

$(1, 2, 4, 2, 4, 4, 6)$

$(0, 2, 2, 2, 4, 4)$

$\{\rho_1 \ \rho_3 \ \rho_4\}$, $\{\rho_5, \rho_6\}$

$\{-1, 0, 1, 1, 2, 2\}$

$(\beta_1\ \alpha_1,\ \beta_2, \alpha_2,\ \beta_3,\ \alpha_3)$

$2 \quad 2 \quad 2$

$\{\rho_1\}\ \{\rho_2\},\ \{\rho_3\ \rho_4\}\ \{\rho_5\ \rho_6\}$

$a^{d_1 + d_2 + 3d_3}$ $(-a^2)^{d_1 + d_2 + d_3}$

$+ 2d_1 + 2d_2 + 2d_3$

$a^{\alpha_1 + \beta_1 + \alpha_2 + \beta_2 + 3\alpha_3 + \beta_3 + 2\alpha_1 + 2\alpha_2 + 5\alpha_3 + \beta_3}$

$= a^{3\alpha_1 + \beta_1 + 3\alpha_2 + \beta_2}$

$q = (-1, -1, 3, 3, 1, 1)$

$\left(8\rho + \frac{3\alpha}{2}\right)^{2n f}$

$(1, -2, 0, -2, -3)$

~~$(3\ 1\ 3\ 1\ 5\ 1)$~~

$\frac{(8\rho + \frac{3\alpha}{2})}{2}^{2nf}$

$\left(4 + \frac{3}{2}\alpha\right)$

$(x q^2\ q^{-2})^r$

$a^2 \to \tilde{a}$

$\frac{\alpha}{2} =$

~~$(1, 2, 1,$~~ $2\ 2$

$\rho_2 = \rho$

$\rho_3 = 2\rho$

$\rho_4 = 2\rho$

$\rho_5 = 3\rho$

$\rho_6 = 3\rho$

	d_1	d_2	d_3	d_4	d_5	d_6
d_1	0	0	1	*	1	*
d_2	0	φ	1	*	1	*
d_3	1	1	2	2	2	2
d_4	*	*	2	2	3	2
d_5	1	1	2	3	3	3
d_6	0	0	2	2	3	3

$\rho_1 = \rho_1$

$\rho_2 = (f+1)\rho$

$\rho_3 = (f+2)\rho$

$\rho_4 = (f+2)\rho$

$\rho_5 = (f+3)\rho$

$\rho_6 = (f+3)\rho$

~~$\rho_1 = \rho_2 = \rho_3 = \rho_4 = \rho_5 = \rho_6 = \rho$~~

$(1, 2, 1, 2, 2, 3)$

~~$\{\boxed{0}\ 1, 1, 2, 2, 2, 2, 3, 3, 3\}$~~

$\{\boxed{0}, 1, 2, 2, 3, 3\}$

$(\boxed{0}, \rho, 2\rho, 2\rho, 3\rho, 3\rho)$ — $\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6$

(ρ_1, ρ)

$$mb + n/b + \frac{Q}{2} - i\left(\frac{iQ}{2} - x_i\right) = mb + n/b + \frac{Q}{2} + \frac{Q}{2} + i x_i = r$$

$$-i^2 = 1 \qquad + v^2$$

$\sum$ T Bar vertex =

~~residue~~ poles

$$x_i = -ib\,m_i - i\frac{n_i}{b}$$

$$e^{-\pi i k_{ij} x_i x_j} = e^{-\pi i k_{ij}\left(ib\,m_i - \frac{in_i}{b}\right)\left(ib\,m_j - \frac{in_j}{b}\right)}$$

$$= e^{-\pi i k_{ij}\left(-b^2 m_i m_j + n_i m_j + n_j m_i - \frac{n_i n_j}{b^2}\right)}$$

$$= e^{-\pi i k_{ij}(n_i m_j + n_j m_i)} \cdot e^{+\pi i k_{ij} m_i m_j b^2}\, e^{\pi i \frac{k_{ij} n_i n_j}{b^2}}$$

when $b \to 0$, $n_i, n_j = 0$

$$\longrightarrow \boxed{e^{+\pi i k_{ij} m_i m_j b^2}} = \boxed{\left(\sqrt{q}\right)^{k_{ij} m_i m_j}}$$

$$e^{+2\pi i \tilde{s}_i x_i} = e^{2\pi i s_i\left(-ib\,m_i - \frac{in_i}{b}\right)}$$

$$= \cancel{e^{2\pi i}}\; e^{2\pi \tilde{s}_i m_i b}\; e^{2\pi \tilde{s}_i n_i / b}$$

when $b \to 0$, $n_i = 0$

$$\longrightarrow \boxed{e^{2\pi \tilde{s}_i m_i b}}$$

$$q = e^{2\pi i b^2}$$

$$S_b\left[\frac{iQ}{2} - x_i\right] = \prod_{m,n \geq 0} \frac{m_i b + \frac{n_i}{b} + Q + i x_i}{m_i b + \frac{n_i}{b} - ix} \xrightarrow{\text{at poles}} m_i b + \frac{n_i}{b} + Q + i\left(-ib\,m_i - i\frac{n_i}{b}\right)$$

$$= m_i b + \frac{n_i}{b} + Q + b\,m_i + \frac{n_i}{b}$$

$$= \prod_{m_i, n_i \geq 0} \left(2 m_i b + 2\frac{n_i}{b} + Q\right)$$

$b \to 0$, $n_i = 0$

$$\longrightarrow \boxed{\prod_{m_i \geq 0}\left(2 m_i b + Q\right)}$$

$$\boxed{x} = e^{-}$$

$$\cancel{x = -ib\,m_i - \frac{n_i}{b}}$$

$x = 0$

$$\frac{S_b\left(\frac{iQ}{2} + ib\,m_i\right)}{S_b\left(\frac{iQ}{2}\right)} \bigg|_{x=0} = \cancel{i^{m_i}\, q^{\frac{m_i(m_i+1)}{2}}}\; \frac{i^{m_i}\left(\sqrt{q}\right)^{\frac{m_i(m_i+1)}{2}}}{\cancel{\prod_{k=1}^{m_i}(1 - q^k)}\;(q, q)_{m_i}}$$

(I)

Vortex = TAW

(II)

$e^{-\pi i k_{ij} x_i x_j} e^{2\pi i \tilde{\zeta}_i x_i} \prod_{i=1}^{N} S_b\left(\frac{iQ}{2} - x_i\right)$

$\rightarrow (\sqrt{q})^{\sum_{ij} k_{ij} m_i m_j} e^{\sum 2\pi b \tilde{\zeta}_i m_i} \prod_{i=1}^{N} \frac{i^{m_i} (\sqrt{q})^{\frac{m_i(m_i+1)}{2}}}{(q,q)_{m_i}}$

$= (\sqrt{q})^{\sum_{ij} k_{ij} m_i m_j + \sum_i \frac{m_i(m_i+1)}{2}} e^{\sum_i (2\pi b \tilde{\zeta}_i + \frac{\pi i}{2}) m_i} \cdot \frac{1}{\prod_{i=1}^{N} (q,q)_{m_i}}$

$= (\sqrt{q})^{\sum_{ij} k^{\text{eff}}_{ij} m_i m_j} e^{\sum (2\pi b \tilde{\zeta}_i + \frac{\pi i b^2}{2} + \frac{\pi i}{2}) m_i} \cdot \frac{1}{\prod_{i=1}^{N} (q,q)_{m_i}}$

$\prod_i x^{m_i}$

$= (\sqrt{q})^{\frac{m_i}{2}} (\sqrt{-1})^{m_i} e^{2\pi b \tilde{\zeta}_i m_i} =$

$\sqrt{q}^{k^{\text{eff}}_{ij} m_i m_j} \left(i q^{\frac{1}{4}} e^{2\pi b \tilde{\zeta}_i}\right)^{m_i}$

$(x, q)_d \sim e^{\frac{1}{\hbar}(\mathrm{Li}_2(x) - \mathrm{Li}_2(q^d x)) + \dots}$

$\sqrt{q} \to -\sqrt{q}$

$\left(-q^{\frac{1}{4}} e^{2\pi b \tilde{\zeta}_i}\right)^{m_i}$

$\tilde{\zeta}_i = \zeta_i - \frac{1}{2} k_{ij} x_j \to \tilde{\zeta}_i = -\zeta_i - \frac{1}{2} k_{ij} x_j$

$-2\pi b \zeta_i \Rightarrow (-)\frac{2\pi b}{2} k_{ij} x_j$

$\log e^{\pi i C_{ij}} (-)^{C_{ij} x_j}$, $n^2 + n$

$\frac{\theta_2}{\theta_1}$ $i\sqrt{q} = \tilde{q}$

$\tilde{q}$, $\sqrt{\alpha} = \tilde{q}$

$q^{\frac{m_i}{4}} = e^{\frac{2\pi i b^2 m_i}{2}}$

$\sqrt{q}^{\frac{m_i}{2}}$

$\sqrt{q}^{k_{ij} m_i m_j + \frac{m_i^2}{2} + \frac{m_i}{2}}$

$= \sqrt{q}^{k^{\text{eff}}_{ij} m_i m_j} \cdot \sqrt{q}^{\frac{m_i}{2}}$

$e^{\sum_i 2\pi b \tilde{\zeta}_i + \frac{i\pi}{2}}$

$(\sqrt{-1})^{m_i} \sqrt{q}^{m_i} = e^{\pi i b^2 m_i}$

$\tilde{\zeta}_i = \zeta_i - \frac{1}{2} k_{ij} x_i$

$b^2 \to b^2 + 1$

$(i^2 \sqrt{q})^{\frac{1}{2}} = i (\sqrt{q})^{\frac{1}{2}}$

$(-\sqrt{q})^{\sum_{ij} k_{ij} m_i m_j + \frac{m_i^2}{2}}$, $(-)^{k_{ij} m_i m_j + \frac{m_i^2}{2}}$

$(-)^{k_{ij} m_i m_j + \frac{m_i^2}{2}} = e^{\frac{\pi i}{2} k_{ij} m_i m_j + \frac{\pi i}{2} m_i^2}$

$(\sqrt{q})^{1}$, $(-\sqrt{q})^{\frac{1}{2}}$

$(\sqrt{-1})^{\frac{1}{2}} q^{\frac{1}{4}}$

$e^{-\pi i b^2 \sum_j k_{ij}}$

$e^{2\pi i b^2} = q$

$1 - b(b + \frac{1}{6})$

$= \cancel{1} - b^2 \cancel{1}$

$(\sqrt{q})^{k_{ij}}$

$-\pi b\, k_{ij} x_j$

(q)

$1 + 1 = 2$

$2 + 4 = 6$

$3 + 9 = 12$

$n(n+1)$

$q = e^{\frac{\hbar}{2} C_{ij} \dots}$

$\pi i C_{ij}$

$(-) = e^{2\pi i}$

$i = e^{\pi i}$

$e^{\frac{\pi i}{2}} = i$

$q \to q\, e^{2\pi i}$

$\sqrt{q} \to -\sqrt{q}$

$i\pi(1 - b(b + \frac{1}{6}))$

$= i\pi(\cancel{1} - b^2 \cancel{1})$

$= -\pi i b^2 \sum_{j=1}^{\text{eff}} k_{ij}$

$$k + \frac{N_c}{2} - \frac{N_{AC}}{2}$$

$\mathbb{Z}_q$-brane.

$$\frac{1}{(x, q^{-1})}$$

$$\begin{array}{ccc} z & Q_1 & Q_1Q_3 \\ \end{array} \quad \left[\begin{array}{ccc} \boxed{0} & -1 & -1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{array}\right] \to \begin{array}{ccc} z & Q_1^{-1} & Q_1,Q_3 \\ \end{array} \left[\begin{array}{ccc} +1 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{array}\right] \to \begin{array}{ccc} z & Q_1 & (Q_1,Q_3)^{-1} \\ \end{array} \left[\begin{array}{ccc} +1 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right]$$

$$\to \begin{array}{ccc} z & Q_1^{-1} & (Q_1Q_3)^{-1} \\ \end{array} \left[\begin{array}{ccc} \boxed{+2} & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right]$$

$\mathbb{Z}_{\bar{f}}$-brane

$$\begin{array}{ccc} z & Q_1 & Q_1,Q_3 \\ \end{array} \left[\begin{array}{ccc} \boxed{1} & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{array}\right] \to \begin{array}{ccc} z & Q_1^{-1} & Q_1,Q_3 \\ \end{array} \left[\begin{array}{ccc} \boxed{0} & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 1 \end{array}\right] \to \begin{array}{ccc} z & Q_1^{-1} & (Q_1Q_3)^{-1} \\ \end{array} \left[\begin{array}{ccc} \boxed{-1} & -1 & -1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{array}\right]$$

$$\begin{array}{ccc} z & Q_1 & (Q_1Q_3)^{-1} \\ \end{array} \left[\begin{array}{ccc} \boxed{0} & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 0 & 0 \end{array}\right]$$

$$(-)^n q^{-\frac{n}{2}}$$

$$\left[\frac{(-1)}{\sqrt{q}}\right]^n$$

$$q^{f\,\kappa/2} \to q^{\frac{f\,n(n-1)}{2}}$$

$$q^{\frac{f}{2}n^2} \cdot q^{-\frac{f}{2}n}$$

$$\left(\frac{1}{\sqrt{q}}\right)^{n+1}$$

$$\left(\frac{1}{\sqrt{q}}\right)^{n(n+1)} x^n$$

$$\left((-)^n q^{\frac{n(n-1)}{2}}\right)^{f+1}$$

$$(-\sqrt{q})^{(f+1)n^2}$$

$j =$

$\tilde{\zeta} \to r - \frac{j}{2} + 1$, $\tilde{r} \to r - \frac{1}{2}$ $\Rightarrow$ $r = \tilde{r} + \frac{1}{2}$

$\tilde{\zeta} = \tilde{r} + \frac{1}{2} - \frac{j}{2} + 1$

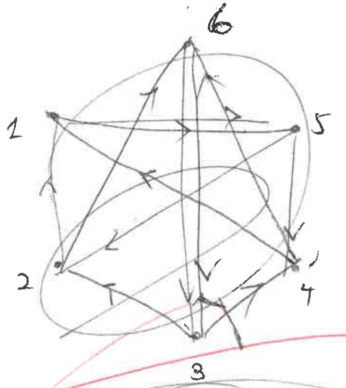
$\boxed{\text{flip} = \text{mutation}}$

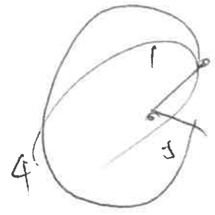
$(-)^{2J+2r} \quad q^{-(J+1)} \quad (\frac{t}{q})^R \quad N_C^{(J,R)}$

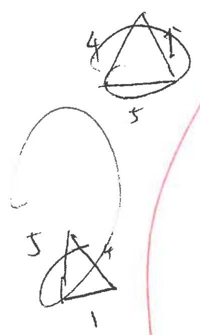
$\begin{pmatrix} 0 & 0 & -1 \end{pmatrix}$

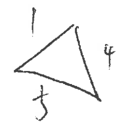
$\frac{1}{2} w_1^2$

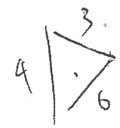
$-\frac{1}{4} w_1 w_7$











four monopole operators ?

internal edges

superpotential W_{3d} $\overset{?}{=}$ twisted eff super $\widetilde{W}_{eff}$ (2d $\#$ k)

glueing condition $\overset{?}{=}$ glueing condition

$\shortparallel$?

characterization ; g. $\overset{?}{\longleftrightarrow}$ 3d $N=2$ Min transf.

symplectic transformation

mixed CS levels and: 3-mfd M_3.

$\boxed{\text{3d} - \text{3d correspondence}}$

~~t-brane~~

t-brane

~~t-brane~~

$$\frac{(-1)^{2s} N_\beta^{(3,r)} q^{-s} t^{r+\frac{1}{2}}}{q^{\frac{1}{2}} - q^{-\frac{1}{2}}} \cdot Q^\beta$$

$$\sum_i [Q_{m_t, i}]_+ \cdot - \sum_i [-Q_{m_t, i}] = Q_{m_i}$$

$$\boxed{\hbar = i\, r b^2}$$

$\cancel{z}^n$

$$\begin{cases} \tilde{\alpha}_i = \alpha_i t^{\frac{1}{2}} q^{-\frac{1}{2}} \\ \tilde{\beta}_i = \beta_i\, t\, q^{-1} \end{cases}$$

$$\begin{cases} \tilde{\alpha}_1 = Q_1 \\ \tilde{\alpha}_2 = Q_1 Q_2 \\ \tilde{\alpha}_3 = Q_1 Q_2 Q_3 \\ \tilde{\beta}_1 = \tilde{Q}_1 \\ \tilde{\beta}_2 = \tilde{Q}_1 \tilde{Q}_2 \\ \tilde{\beta}_3 = \tilde{Q}_1 \tilde{Q}_2 \tilde{Q}_3 \end{cases} \Rightarrow \begin{cases} \boxed{Q_1 = \tilde{\alpha}_1 = \alpha_1 t^{\frac{1}{2}} q^{-\frac{1}{2}}} \\ Q_2 = \tilde{\alpha}_2 \tilde{\alpha}_1^{-1} = \alpha_2/\alpha_1 \\ Q_3 = \tilde{\alpha}_3 \tilde{\alpha}_2^{-1} = \alpha_3/\alpha_2 \\ \boxed{\tilde{Q}_1 = \tilde{\beta}_1 = \beta_1 t q^{-1}} \\ \tilde{Q}_2 = \tilde{\beta}_2 \tilde{\beta}_1^{-1} = \beta_2/\beta_1 \\ \tilde{Q}_3 = \tilde{\beta}_3 \tilde{\beta}_2^{-1} = \beta_3/\beta_2 \end{cases}$$

$$\alpha_i = \tilde{\alpha}_i$$

$$[3]_+ [-3]_+$$

$$\frac{\tilde{\alpha}_2}{\tilde{\alpha}_1} = \frac{\alpha_2}{\alpha_1}$$

$Q_{m,w} \rightarrow 0$

$$e^{-\frac{x a}{b \hbar^2}}$$

$$z'(t)\, z''(t)$$

$$\begin{cases} z'(t) = 2\left[-u_{m_t}(t) - u_{m_t}(t+1) + \sum_i [Q_{m_t, i}(t)]_+ u_i(t) \right] \\ z''(t) = 2\left[u_{m_t}(t) + u_{m_t}(t+1) - \sum_i [-Q_{m_t, i}(t)]_+ u_i(t) \right] \end{cases}$$

$$\sum_w [Q_{m,w}]_+ + \sum_w [-Q_{m,w}]_+ .$$

$$\vec{m} = (m_1, m_2, \dots m_L),$$

$$\begin{cases} z'(m) = 2\left[-x^{(m)} - x'^{(m)} + \sum_w [Q_{m,w}]_+ x_w \right] \\ z''(m) = 2\left[x^{(m)} + x'^{(m)} - \sum_w [-Q_{m,w}]_+ x_w \right] \end{cases}$$

$$\sum_w [Q_{m,w}]_+ (x^{(m)} + x'^{(m)}) \cdot x \rightarrow 0,$$

$$\sum_m \frac{z'(m)\, z''(m)}{4} = \cancel{-x^{2(m)} - x'^{(m)}} - x^2(m) - 2x'(m)x(m) - x'^2(m) + \cancel{x(m)} \sum_w [Q_{m,w}]_+ x_w + \cancel{x'(m) \sum_w [Q_{m,w}]_+ x_w} - \sum_{w_1} \sum_{w_2} [Q_{m,w_1}]_+ [-Q_{m,w_2}]_+ x_{w_1} x_{w_2}$$

$$= \sum_m \left[-x^2(m) - 2x'(m)x(m) - x'^2(m) \right.$$

$$+\, x(m) \sum_w [-Q_{m,w}]_+ x_w + x'(m)[-Q_{m,w}]_+ x_w$$

$$\sum_m (x(m) + x'(m)) \sum_w |Q_{m,w}|\, x_w .$$

- $\tilde{T}_1$ theory w/ $\tilde{w} \neq 0$ = $\tilde{T}_2$ w/ ~~without~~ $\tilde{w} \doteq 0$

程实

- superpotential of gluing $W_{super} \neq 0$ $\longleftrightarrow$ CS levels k_{ij}^{eff}, w/ $W_{super} = 0$
- U(N) type of mirror symmetry, Abelianization
- T-branes $\leftrightarrow$ monopole operators, $\leftrightarrow$ k_{ij}^{eff}
- $\tilde{W}_{eff} \overset{?}{=} \mathrm{Vol}(M_3)$ and how to get M_3 from ~~Weff~~, k_{ij}^{eff}

- T-brane ⇔ 3d mirror symmetry
- M2 ; CY4 ⇔ 3d mirror symmetry + mixed CS levels
- Abelianization ; U(N) — 3d mirror symmetry
- seiberg duality + abelianization + 3d mirror symmetry
- higher form symmetry in 3d $N=2$
- string junction ⇔ 3d $N=2$

? how to produce mixed CS level generically ?

solutions of susy ground states $\overset{?}{=}$ # phases or ~~Mirror~~ mirror sym graph = $H(Q)$

given ~~a~~ 3d $N=2$, what is the 3-mfd M_3, ; where 3d $N=2$ = $T[M_3]$? higher form sym ?

- Quiver reduction reliable ?

- Modeli space

110 cm × (< 160) cm

- different positions of Lag-brane (phase) toric CY3.
- complete intersection realization of $(① - \boxed{1})^N_{k_{ij}}$ moduli space

condition ? ~~no phase tro~~ wall crossing

another way of geometric realization ?

refined open

- blow-up eqns
- YY function from strip geometry.

refined algebraic realization of refined open.

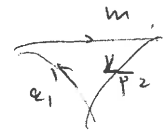
~~top vertex~~

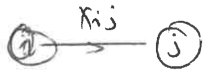
$$\delta \int_{C_3} F_2 = \#(C_2 \cap D6)$$

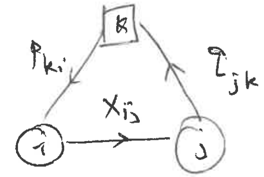
$$\delta \int_{C_4} F_4 = \#(C_4 \cap D6 \cap C_4^{(Fvv)})$$

$$p_1 X_1 q_1 + p_2 X_2 q_2 \qquad \delta W = m\, p_2 q_1$$

$$= \cancel{p_1 (X_1 q_1}$$







$$X_{ij} = 0 \quad \Rightarrow \quad \delta W = p_{ki} X_{ij} q_{jk}.$$

$$T,\ \tilde{T}, \qquad U(1)^G$$

$$T^{(n)}\ T^{(-n)} = \left(\prod_a X_a^{h_a}\right)^{|n|}$$

$$p_1 X_1 q_1 + p_2 X_2 q_2 + m p_2 q_1$$

$$\cancel{p_1 X_1 q} \quad = \cancel{p_1 + m}$$

$$= (p_1 X_1 + m p_2) q_1 + p_2 X_2 q_2,$$

$$= p_1 X_1 q_1 + \cancel{p_2 X_2 + m} p_2 (X_2 q_2 + m q_1)$$

$$\frac{\partial W}{\partial X_a} = \frac{\partial W_0}{\partial X_a} + \Sigma p_a q_a = 0$$

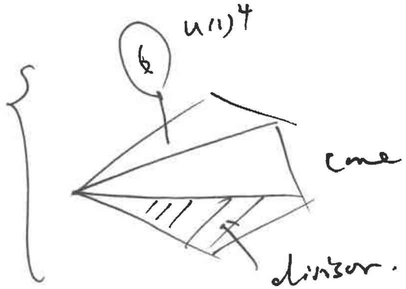
M2 probe theory Y_4

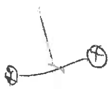
Y_4 M-theory

$\downarrow$ U(1) isometry ($\mathbb{R}$)

IIA ~~theory~~ string

isometry $U(1)^4$,





$e^{-2b\pi s} = \Sigma$

$e^{\frac{3\pi i}{2} d_1}$

$e^{\pi i d_1 d_2} = \boxed{(-)^{d_1 d_2} \, e \, (i)^{k d_1^2}} \, (-i)^{d_1}$

$e^{\frac{\pi i}{2} d_1^2 k} \equiv (-i)^{k}$

$\frac{\log \Sigma}{-2b\pi} = s$

$e^{b\pi n_i} = Y_i$

$q = e^{2\pi i b^2} = e^{2\pi i b Q}$

$e^{\frac{\pi i}{2}} = i$, $e^{\frac{\pm \pi i}{2}} = -1$

$x_i = (-)^{k_{ii}^{\text{eff}}} e^{\xi_i^{\text{eff}}}$

$\left[\frac{\log[Y_i]}{b\pi}\right] = n_i$

$\frac{\log[q]}{2\pi i b} = \cancel{2\pi i b Q}$

$h\,D6$

h

worldvolume of D2 probes

$(-\sqrt{q})^{k_{ij}^{\text{eff}} d_i d_j} \frac{x_1^{d_1} x_2^{d_2}}{(q,q)_{d_1} (q,q)_{d_2}}$

$\cancel{(Q, q^{\ldots}) = (Q)}$

$i^{d_1^2}$, $(-)^{d_1 d_2}$, $i^{d_2^2}$, $i^{k d_1^2}$,

$= (-)^{d_1 d_2} \, i^{k d_1^2} \, i^{d_1^2 + d_2^2}$

$\mathbb{Z}_{\text{het}}$ ν.

CS th

$\mathbb{F}_2 \to$

RR-flux

mfd-7

$CY_3 = CY_4 / U(1)_M$

$CY_3 =$

D2, RR-fl., $\mathbb{F}_2 \to$ CS terms.

D6 : collection of ~~toric~~ toric divisors

D2 on D6 : : massless fermions, bifund X

$SW = p \times q$

$\supset$ D6

CY_3, CY_4,

2d toric cone

3d toric cone

dimension reduct

$N=1$ 4d-theory

$N=2$ 3d theory

IIA $4+6=10$, $3+8=11$ M-theory

M2 / M-theory $\to$ D2 / D6 IIA.

D6 — flavors

in D6 flavor sym SU(N)

SU(N)